A LINEAR ADDITIVE MULTI-ATTRIBUTE UTILITY LOCATION MODEL

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Abstract: This paper elaborates the development of a linear additive multi-attribute utility location model. A survey of past location studies was initially conducted to investigate the numerous models location analysts had employed in pinpointing the best site. Characteristics of past location models were subsequently analyzed and classified to identify their advantages and disadvantages, which were used as the bases for formulating the linear additive multi-attribute utility location model. The model assures quality of strategic decision-making on site selection via the multi-criteria evaluation of candidate locations. The prime strength of the model lies in its ability to incorporate subjective, judgmental inputs from management with objective, factual location data to recommend the best facility location.

INTRODUCTION
Facility location decisions are of major significance to the company’s competitive position. Suitability of a site rests on how well those involved in the decision-making process assess the impact that a site will have on the fulfillment of corporate objectives. A suitable location can provide favorable contributions to a company’s market competitiveness. These include monetary and non-monetary contributions, such as increase in production capacity, additional profit, business expansion, better service to customers, increase in stockholders’ wealth, cost reduction, and shortened manufacturing lead time, etc. On the other hand, an unsuitable location can bring about adverse effects to the firm. The new facility must be carefully evaluated and located to obtain strategic advantages over competitors.

Conventional approaches to solving location problems require the gathering and analysis of voluminous amounts of information on location alternatives and identification of the site, which best fits the organization’s needs. Facility location models, ranging from simple scoring models to more sophisticated techniques, such as mathematical programming, decision analysis, and regression analysis have been used to facilitate data analysis [18]. The decision-making process consumes much time and effort, involves consideration of a large number of both quantitative and qualitative factors, and requires subjective evaluation and managerial judgment. High quality decisions therefore require extensive discussion and debate and demand significant amount of time and effort before a consensus on the best location is secured.

In this paper, the development of a linear additive multi-attribute utility location model, which can assist facilities planners in obtaining quality solutions to facility location problems, is described. The model entails the processing of a vast amount of information on location alternatives and eventually identifies the most suitable sites.

LITERATURE REVIEW
Several location models have been proposed based on various sets of assumptions and are thus suitable for different applications. Taxonomy of location models would include the following categories:

• Manual models, requiring objective location factors, e.g. Von Thunen’s Theory of Location [22], Weber’s Theory of Location [23], Hoover’s Cost Analysis [7], Interdependence Theory of Location [3] [8], The Theory of the Maximum Profit Location [12], Plant Location on a Straight Line [2], and Center of Gravity Models [25].
- **Manual models**, requiring **objective** or **subjective** location factors, e.g. Ranking (Scoring) Models [9] [14] [17] [18] [21], and Dimensional Analysis [20] [25].
- **Computer models**, requiring **objective** location factors, e.g. Plant Location in a Transportation Network [2], Plant Location with Respect to Rectangular Transportation Movements [2], Plant Location by Vector Summation [2], Plant Location by Iterative Method [2], Mathematical Programming [5] [13], and Regression Analysis [18].
- **Computer models**, requiring **objective** or **subjective** location factors, e.g. Decision Analysis [11] [16], and Multi-attribute Utility Models [1].

Location models are based on a range of objectives, including cost minimization, profit maximization, and the optimization of qualitative factors. Manual location models that were formulated in the beginning of location studies involved simple calculations whose solution did not require the use of computers. The advent of computers has enabled the use of location models with complex mathematical formulations. The choice of a particular model does not depend on its technical sophistication, but rather on its relevance to a particular application.

Examples of computer-based location models are Expert Choice [4], which utilizes Saaty’s Analytic Hierarchy Process (AHP) and Automan [24], which incorporates a modified AHP procedure. Both of these approaches require the development of a logical hierarchy of location factors and utilize pairwise comparisons to establish the appropriate weight to be assigned to each factor.

**LOCATION MODEL FORMULATION**

A linear additive multi-attribute utility location model is proposed for evaluation of location alternatives. The approach does not automatically eliminate location alternatives, which do not meet an essential threshold. Instead, it presents the analyst with a ranked shortlist of good alternatives based on the entire range of location factors and ratings provided. The following notations are adopted.

\[
\begin{align*}
\text{m}_a & \quad \text{Number of published data representing location subfactor a.} \\
\text{n}_b & \quad \text{Number of subfactors of location factor b.} \\
\text{o} & \quad \text{Number of location factors.} \\
\text{p} & \quad \text{Number of location alternatives.} \\
\text{l}_k & \quad \text{Location alternative k.} \\
\text{x}_{hij}(l_k) & \quad \text{Published data h representing subfactor i of factor j in location k.} \\
\text{X}_{hij}(l_k) & \quad \text{Normalized published data h representing subfactor i of factor j in location k.} \\
\text{population(l}_k) & \quad \text{Resident population in location k.} \\
\text{area(l}_k) & \quad \text{Area of location k.} \\
\phi_{hij}(l_k) & \quad \text{Scaled published data h representing subfactor i of factor j in location k.} \\
\phi_{ij}(l_k) & \quad \text{Subfactor score for subfactor i of factor j in location k.} \\
\phi_{j}(l_k) & \quad \text{Factor score for factor j in location k.} \\
\phi_{k} & \quad \text{Score for location k.} \\
\alpha & \quad \text{Range factor.} \\
\beta & \quad \text{Range offset value.} \\
U_{hijk} & \quad \text{The utility function for published data h representing subfactor i of factor j in location k.} \\
U_{ijk} & \quad \text{The utility function for subfactor i of factor j in location k.} \\
U_{jk} & \quad \text{The utility function for factor j in location k.} \\
U_{k} & \quad \text{The utility function for location k.} \\
W_{i} & \quad \text{Weight of subfactor i.} \\
W_{j} & \quad \text{Weight of factor j.} \\
F & \quad \text{The function for aggregating values of utility functions.} \\
N & \quad \text{The number of recommended locations.} \\
\tau & \quad \text{The threshold level.}
\end{align*}
\]


The procedure employed in formulating the location model used in the system is outlined below:

**Step 1: Translate published location data to subfactor scores.**

The magnitude of published location data \( x_{hi}^{(l)}(k) \) may depend on population. For instance, a location with more resident population tends to have more housing units than the one with less resident population. In addition, the magnitude of certain data, such as parks and recreation areas may vary in proportion to location areas. There are published data that can be used directly, such as unemployment rates and crime rates. Thus, published location data need to be normalized before they can be used in the evaluation. Normalized published data, \( X_{hi}^{(l)}(k) \), are computed using Equations (1), (2), and (3):

\[
X_{hi}^{(l)}(k) = \frac{x_{hi}^{(1)}(k)}{\text{population}(1_k)} \quad (1)
\]

for population-dependent location data, or

\[
X_{hi}^{(l)}(k) = \frac{x_{hi}^{(1)}(k)}{\text{area}(1_k)} \quad (2)
\]

for area-dependent location data, or

\[
X_{hi}^{(l)}(k) = x_{hi}^{(l)}(k) \quad (3)
\]

for independent location data.

Published location data may have different units. Therefore, to facilitate comparison, they must be scaled so that their values fall within the same range. Another important consideration in scaling lies in the relationship between magnitudes of published data and their merits. For example, some data are better than others if their values are larger, and vice versa. Scaling can thus be accomplished by using Equation (4) and Equation (5) for published location data with direct and inverse relationship, respectively.

\[
\phi_{hi}^{(l)}(k) = \alpha \left[ \frac{X_{hi}^{(1)}(k) - \min\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\}}{\max\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\} - \min\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\}} \right] + \beta \quad (4)
\]

for direct relationship.

\[
\phi_{hi}^{(l)}(k) = \alpha \left[ 1 - \frac{X_{hi}^{(1)}(k) - \min\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\}}{\max\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\} - \min\{X_{hi}^{(1)}, X_{hi}^{(12)}, \ldots, X_{hi}^{(1p)}\}} \right] + \beta \quad (5)
\]

for inverse relationship.

Let \( A(g, k) = \)

\[
\left[ \frac{g^{(1)}(k) - \min\{g^{(1)}, g^{(12)}, \ldots, g^{(1p)}\}}{\max\{g^{(1)}, g^{(12)}, \ldots, g^{(1p)}\} - \min\{g^{(1)}, g^{(12)}, \ldots, g^{(1p)}\}} \right] \quad (6)
\]
Equations (4) and (5) can be rewritten as shown in Equations (7) and (8), respectively.

\[
\begin{align*}
\phi_{hij}(l_k) &= \alpha[A(X_{hij}, k)] + \beta \quad \text{...for direct relationship.} \\
\phi_{hij}(l_k) &= \alpha[1-A(X_{hij}, k)] + \beta \quad \text{...for inverse relationship.}
\end{align*}
\]

For this model, the scaled published data will be assigned values in the range of 1 to 10, as suggested by Muther and Hales [15]. Hence, \(\alpha\) and \(\beta\) will be set at 9 and 1, respectively.

Next, scaled published data will be used in the multi-attribute utility model, shown in Equation (9), for computing subfactor scores.

\[
U_{ijk}[\phi_{1ij}(l_k), \phi_{2ij}(l_k), \ldots, \phi_{m_{ij}}(l_k)] = f[U_{lijk}[\phi_{1ij}(l_k)], \ldots, U_{m_{a}ijk}[\phi_{m_{a}ij}(l_k)]]
\]

According to Keeney and Raiffa [10], \(f\) can simply be defined as a linear additive function, thus

\[
f[U_{lijk}[\phi_{1ij}(l_k)], \ldots, U_{m_{a}ijk}[\phi_{m_{a}ij}(l_k)]] = \sum_{h=1}^{m_a} U_{hijk}[\phi_{hij}(l_k)].
\]

The results obtained from Equation (10) will be scaled to have values within the same range of 1 to 10 by using Equation (11).

\[
\phi_i(l_k) = \alpha[A(U_{ijk}, k)] + \beta,
\]

where \(U_{ijk} = U_{lijk}[\phi_{1ij}(l_k), \phi_{2ij}(l_k), \ldots, \phi_{m_{ij}}(l_k)]\).

**Step 2: Compute factor scores from subfactor scores and user’s inputs.**

Subfactor scores will be used in the linear multi-attribute utility model, shown in Equation (12), for computing factor scores.

\[
U_{jk}[\phi_{1j}(l_k), \phi_{2j}(l_k), \ldots, \phi_{n_{bj}}(l_k)] = f[U_{lijk}[\phi_{1ij}(l_k)], \ldots, U_{n_{bj}jk}[\phi_{n_{bj}j}(l_k)]]
\]

where \(f[U_{lijk}[\phi_{1ij}(l_k)], \ldots, U_{n_{bj}jk}[\phi_{n_{bj}j}(l_k)]] = \sum_{i=1}^{n_b} w_i U_{lijk}[\phi_{ij}(l_k)]\).

\(w_i\) can be 0 or 1 and is assignable by users. If \(w_i\) is assigned the value of 0, subfactor \(i\) will not be considered in the evaluation. On the other hand, subfactor \(i\) will be evaluated if \(w_i\) is set to 1. Results from Equation (12) are the total scores of location factor \(j\) in location \(k\). They will be scaled to have values within the same range of 1 to 10 by using Equation (13).

\[
\phi_j(l_k) = \alpha[A(U_{jk}, k)] + \beta,
\]

where \(U_{jk} = U_{lijk}[\phi_{1ij}(l_k), \phi_{2j}(l_k), \ldots, \phi_{n_{bj}}(l_k)]\).

**Step 3: Calculate location scores from factor scores and user’s inputs.**

\(\phi_j(l_k)\) calculated from Equation (13) will subsequently be used in the linear additive multi-attribute utility model for computing the total score of location \(k\), as shown in Equation (14).

\[
U_k[\phi_1(l_k), \phi_2(l_k), \ldots, \phi_\alpha(l_k)] = f[U_{1lk}[\phi_1(l_k)], \ldots, U_{ok}[\phi_\alpha(l_k)]],
\]

where \(f[U_{1lk}[\phi_1(l_k)], \ldots, U_{ok}[\phi_\alpha(l_k)]]] = \sum_{i=1}^{\alpha} W_j U_{jk}[\phi_j(l_k)]\).
W<sub>j</sub> is the factor weight or the level of importance which represents the user’s judgement on the relative importance of location factor j. W<sub>j</sub> can be assigned by adopting the widely-accepted vowel-letter rating scheme [15], i.e.

<table>
<thead>
<tr>
<th>Level of Importance</th>
<th>Equivalent Numeric Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely important</td>
<td>4</td>
</tr>
<tr>
<td>Essentially important</td>
<td>3</td>
</tr>
<tr>
<td>Important</td>
<td>2</td>
</tr>
<tr>
<td>Ordinarily important</td>
<td>1</td>
</tr>
<tr>
<td>Unimportant</td>
<td>0</td>
</tr>
</tbody>
</table>

This approach is based on the point allocation method, which was found to work very well when compared to other methods of determining weights in additive utility models [19]. The choice of the weighting method is itself a multi-criteria one, involving consideration of ease of use, mean performance, dispersion, normative justification, and trustworthiness.

Equation (14) will provide the total score of location k. This will be scaled to have values in the range of 0 to 100 by using Equation (15), with the best location receiving a score of 100.

\[
\phi(l_k) = 100[A(U_k, k)], \quad (15)
\]

where \( U_k = U_k[\phi_1(l_k), \phi_2(l_k), \ldots \phi_o(l_k)] \).

Equations (1) to (15) will be calculated for all location scores by varying values of variables, i.e.

\[
\begin{align*}
  h &= 1, 2, \ldots, m_a, \\
  i &= 1, 2, \ldots, n_b, \\
  j &= 1, 2, \ldots, o, \text{ and} \\
  k &= 1, 2, \ldots, p.
\end{align*}
\]

For this model, \( m_a \) varies from subfactor to subfactor. Similarly, \( n_b \) differs from one factor to the other. The number of location factors, \( o \), and the number of location alternatives, \( p \), depend on user’s selection. When all location scores are computed, they will be compared and the top few locations will be recommended. There is no specific requirement concerning an appropriate number of locations to be recommended. A review of literature was conducted to determine the number of candidate locations that had been used by companies for detailed evaluation [6], [15], [20]. Results from the literature review suggested that the number of recommended locations typically ranged from 3 to 10. Equation (16) is used to select the desired number of locations.

\[
N = \begin{cases} 
3, & \text{if } i \leq 3, \\
3, & \text{if } 3 < i < 10, \text{ and } x/y > \tau, \\
i, & \text{if } 3 < i < 10, \text{ and } x/y \leq \tau, \\
10, & \text{if } i \geq 10,
\end{cases} \quad (16)
\]

where \( i = 1, 2, 3, \ldots \) 
\( x \) = \( i^{th} \) rank location score, and
\( y \) = \( 3^{rd} \) rank location score.

The threshold level will be set such that the desired number of locations is recommended.

**CONCLUSION**

The linear additive multi-attribute utility location model has been successfully formulated. This novel approach can be readily adopted to facilitate informed decisions on any types of facilities such as offices, factories, service industries, hazardous waste sites, and recreational facilities. In this research, the model was proposed primarily to facilitate manipulation of the varied and extensive amounts of subjective and objective location data required for effective decision-making. While this paper focuses on the conceptual description of the linear additive multi-attribute utility location model, its implementation involves the evaluation of several factors and the determination of appropriate weights for each factor. The model provides a structured approach to location selection, ensuring that the final decision is based on a comprehensive assessment of various criteria. By systematically combining the evaluation of multiple factors, the model aims to deliver the most suitable location option. Future work could involve the development of additional factors and the refinement of the weighting scheme to better reflect the complexity of real-world decision-making processes.
model, further work is planned to validate its applicability through actual implementations. The ultimate goal is the development of a robust location model, which may be adopted to assist location analysts in arriving at good decisions reliably and practically.

REFERENCES