An Optimal Production Control Policy for an Unreliable Machine System with Buffer

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Abstract

The paper is to present a simulation method to search for an optimal (z, Z) (z < Z) values for the (z, Z) production control policy so that the expected inventory cost rate (i.e. expected total cost due to both inventory holding and backlog per unit time) is minimal for a single machine system with buffer. The system is subject to operation-dependent failure only and alternates between normal and failure. An algorithm is presented to illustrate how to estimate the inventory cost rates for various pairs of (z, Z) and how to get the minimal one among them.

Notations

C^+	holding cost per unit time for a single product
C^-	shortage cost per unit time for a single product
d	demand rate (constant)
γ	maximum machine production rate
$u(t)$	machine production rate at t
$J(t)$	the inventory level of the system at t
$X_1, X_2,...$	i.i.d. Up times of the machine
$Y_1, Y_2,...$	i.i.d. Down (i.e repair) time of the machine
λ	the reciprocal of mean up time, i.e. λ=1/MTTF
μ	the reciprocal of mean down time, i.e. μ=1/MTTR
(z, Z)	inventory lower and upper control levels for the (z, Z) production control policy

1. Introduction

Production control policies are mainly to control/adjust the machine production rate so that the scheduled customers' demands can be satisfied in a most cost-effective way, which is to minimize a chosen objective function, expressed in general as a cost function in terms of machine reliability and capacity, inventory holding, backlog and etc.. Akella and Kumar (1986) proposed the so-called HP (Hedge Point) production control policy for a single machine system to control/adjust machine production rate to keep the inventory as close to the optimal level z^* as possible based on the belief that such a z^* exists (cf. Bai and Gershwin (1990, 1995), Mbihi and Malhame (1999), and Shairfnia (1998), *etc.*). Bielecki and Kumar (1998), and Fong and Zhou (2000) proposed the so-called Zero-Inventory control policy to control/adjust production rate to keep the inventory as close to zero based on the belief that $z^* = 0$ in certain special conditions. Bielecki and Kumar (1998) proved that the zero-inventory control policy could indeed minimize the inventory cost rate in such conditions. Ching (1998) proposed the (z, Z) production control policy to control/adjust production rate according a rule in terms of two-control limits $z < Z$ on inventory level. He mentions without proof that HP policy can be a special (z, Z) policy in which $z^* = Z = z + 1$ where z^* is the optimal level in HP policy.

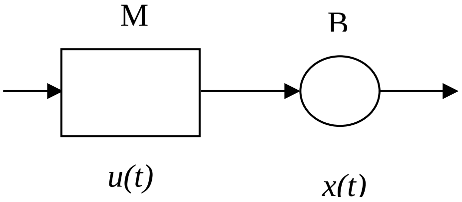
The purpose of this article is to further study the (z, Z) policy in a way to search for (z, Z) values, which minimize the expected inventory cost rate (i.e. the expected total cost due to both inventory holding and backlog per unit time) by a simulation method for a single unreliable machine manufacturing system. This simulation is based on the fact that the machine is subject to operation-dependent failure (i.e. it deteriorates and then fails due to operation only) and alternates between Normal and Failure.

This paper is organized as follows. In section 2, the system assumptions and the (z, Z) production control policy are listed, and several mathematical results which are used to calculate/estimate the expected inventory cost rate for the system under such a policy are then presented. In section 3, the procedure for the proposed simulation method is introduced. In section 4, a numerical example is presented to illustrate the simulation method, and the results will be compared with those by eM-plant software. In section 5, some concluding remarks and future studies will be presented.

2. System Assumptions and (z, Z) Production Control Policy

Throughput of this paper, the system consists of a single unreliable machine with final buffer, as shown in Fig. 1. It further satisfy the following assumptions:

1) The machine is to produce a single type of products.
2) The machine alternates between Normal and Failure such that it follows the alternating renewal process $\{X_n , Y_n\}_{n\geq 1}$ in which X_n &Y_n are the n-th Up and Down times of the machine respectively.
3) The machine, whenever it is normal, can produce products at any rate μ from 0 to γ where γ denotes the maximum production rate.
4) The machine subject to operation-dependent failures only (i.e. its failure is due to operation only). In particular, the machine won't fail whenever its production rate is zero.
5) The machine production rate will be zero whenever it is failed.
6) The demand rate d for the product is assumed to be constant.



Note: M=machine, and B=buffer

Figure 1. An unreliable single machine with buffer

2.1 A (z, Z) Production Control Policy

Let $J(t)$ be the inventory level of the system at t. Then the system is said to have inventory (resp. backlog) $J(t)$ at t if $J(t) > 0$ (resp. $J(t) < 0$). In general, $J(t) = J^+(t) - J^-(t)$

where $J^{+}(t)=\max\{J(t),0\}$ and $J^{-}(t)=\max\{-J(t),0\}$.

Under the (*z*, *Z*) production control policy, the machine speed is controlled according to the rule *J*(*z*, *Z*) [10] given as follows：

1) The machine starts at its maximal capacity γ at $t = 0$ with *J*(0)=0 until either the inventory level reaches *Z* or machine fails whichever occurs first.

2) Once *Z*-level is reached, the machine production rate will be re-set to zero until the inventory level reaches *z*-level (machine failure won't happen during this period).

3) Once *z*-level is reached, the machine production rate will be re-set to γ until the inventory level reaches *Z*-level or machine fails whichever occurs first.

4) Whenever the machine fails, the machine production rate will be zero until its restoration to Normal. Upon each restoration, the machine production rate will be set to γ until the inventory level reaches *Z*-level or machine fails whichever occurs first.

Let a_n and b_n be the times to the *n*th failure and *n*th restoration of the machine respectively, and $A_n = J(a_n)$ & $B_n = J(b_n)$ the inventory levels of the system at a_n and b_n respectively for each *n*=1,2,3,…. Then

Lemma 1: Given the machine in the alternating renewal process $\{X_n, Y_n\}_{n\ge 1}$, then for each *n*=1,2,3,…,

$$a_n = b_{n-1} + X_n \quad \& \quad b_n = a_n + Y_n \tag{1}$$

Furthermore, if the system is under the (*z*, *Z*) production control policy, then

$$A_n = B_{n-1} + X_n(r-d) \,\&\, \; B_n = A_n + Y_n(-d) \tag{2}$$

if $a_0 = b_0 = 0$

Proof: Eq. (1) is obvious. Eq. (2) follows simply according to the above rule *J*(*z*, *Z*).

2.2 The Mathematical Results Used To Estimate the Expected Inventory Cost Rate

It is observed that in the stochastic process $\{J(t)\}_{t\ge 0}$, which is based on that the machine is in the alternating renewal process $\{X_n, Y_n\}_{n\ge 1}$, a renewal is to occur whenever the inventory level reaches *Z*. For convenience, let T_Z denote the time between successive renewals. Then, by Theorem 3.6.1 in Ross (1993) but with minor manipulation, the expected inventory cost rate *H* (i.e. the expected total cost due to inventory holding and backlog per unit time) in the long run will be as follows;

$$H=\lim_{T\to\infty}\frac{1}{T}E\int_0^T (C^+J^+(t)+C^-J^-(t))dt=\frac{1}{E(T_Z)}E[\int_0^{T_Z}(C^+J^+(t)+C^-J^-(t))dt] \tag{3}$$

To estimate *H*, we may assume without losing generality that $J(0)=Z$ (i.e. the system starts with inventory level *Z* rather than 0 at *t* = 0) for simplicity. Then such estimation is reduced to estimate both $E(\boldsymbol{T}_Z)$ and $E[\int_0^{TZ}(C^+J^+(t)+C^-J^-(t))dt]$ as follows;

$$E(T_Z)=\sum_{k=0}^{\infty}E(T_Z \mid N(T_Z)=k)P\{N(T_Z)=k\} \tag{4}$$

$$E\int_0^{T_Z}(C^+J^+(t)+C^-J^-(t))dt$$

$$=\sum_{k=1}^{\infty}E[\int_0^{T_Z}(C^+J^+(t)+C^-J^-(t)) \mid N(T_z=k)]dt\cdot P\{N(T_z)=k\} \tag{5}$$

in which T_Z may be regarded as the time to the first renewal.

Under the assumption that *J*(0)=*Z*, the machine production rate starts at zero and so $J(t)$ decreases according to $J(t)=Z-dt$ from *Z* to *z* during [0, $\frac{Z-z}{d}$]. Hence, we may let

$$a_0 \equiv 0 \;\; \& \;\; b_0 \equiv \frac{Z-z}{d} \tag{6}$$

$$A_0 = J(a_0) = Z \tag{7}$$

$$B_0 = J(b_0) = z \tag{8}$$

Proposition 1: Given the machine in the alternating renewal process $\{X_n, Y_n\}_{n\geq 1}$ and under the (z, Z) production control policy but with $J(0)=Z$, then for each n=1,2,3,…,

$$a_n = \frac{Z-z}{d} + \sum_{i=1}^{n-1}(X_i + Y_i) + X_n \quad \&$$

$$b_n = \frac{Z-z}{d} + \sum_{i=1}^{n}(X_i + Y_i) \tag{9}$$

$$A_n = z + \sum_{i=1}^{n-1} X_i(r-d) - \sum_{i=1}^{n-1} Y_i d + X_n(r-d) \tag{10}$$

$$B_n = z + \sum_{i=1}^{n} X_i(r-d) - \sum_{i=1}^{n} Y_i d \tag{11}$$

Proof: It is known that $a_1 = \frac{Z-z}{d} + X_1$ (as the machine starts with a non-zero production rate since $\frac{Z-z}{d}$) and so $b_1 = a_1 + Y_1 = \frac{Z-z}{d} + (X_1 + Y_1)$. Suppose that $a_n = \frac{Z-z}{d} + \sum_{i=1}^{n-1}(X_i + Y_i) + X_n$. Then by Eq. (1), $b_n = a_n + Y_n = \frac{Z-z}{d} + \sum_{i=1}^{n}(X_i + Y_i)$ and so $a_{n+1} = \frac{Z-z}{d} + \sum_{i=1}^{n}(X_i + Y_i) + X_{n+1}$. Hence, Eq. (9) is concluded. Suppose that $A_n = z + \sum_{k=1}^{n-1} X_k(r-d) - \sum_{k=1}^{n-1} Y_k d + X_n(r-d)$. Then by Eq. (2), we obtain that

$$B_n = A_n + Y_n(-d) = z + \sum_{k=1}^{n-1} X_k(r-d) - \sum_{k=1}^{n-1} Y_k d + X_n(r-d) + Y_n(-d)$$

$$= z + \sum_{k=1}^{n} X_k(r-d) - \sum_{k=1}^{n} Y_k d$$

and

$$A_{n+1} = B_n + X_{n+1}(r-d) = z + \sum_{k=1}^{n+1} X_k(r-d) - \sum_{k=1}^{n} Y_k d$$

Hence, Eqs. (10) and (11) are concluded.

q.e.d.

Proposition 2: Given the machine in the alternating renewal process $\{X_n, Y_n\}_{n\geq 1}$ and under the (z, Z) production control policy but with $J(0)=Z$, then for each n= 0,1,2,…,

$$J(t) = \{ \begin{matrix} A_n - d(t-a_n) & if \quad t \in [a_n, b_n] \\ B_n + (r-d)(t-b_n) & if \quad t \in [b_n, a_{n+1}] \end{matrix}$$

Proof: During [a_0, b_0]=[0, $\frac{Z-z}{d}$], the machine production rate will be zero and so the inventory level will decrease from Z until z according to $J(t) = Z - dt = A_0 - d(t-a_0)$. During[$b_0$, a_1], the inventory will increase from z to A_1 according to $J(t) = z + (r-d)(t - \frac{Z-z}{d})$. At the nth failure time a_n, the inventory level will be A_n and the production rate will be zero until b_n. Hence, during [a_n, b_n], $J(t) = A_n - d(t-a_n)$. At the nth restoration time b_n, the inventory level will be B_n and the production rate will be γ until a_{n+1}. Hence, during [b_n, a_{n+1}], $J(t) = B_n - (\gamma - d)(t-b_n)$.

q.e.d.

Proposition 3: Given $N(\boldsymbol{T}_Z)=k$ *and* $J(0)=Z$,

$$J(t) = (Z-dt)I_{[0,\frac{Z-z}{d}]}(t) + z + (r-d)(t - \frac{Z-z}{d})I_{[\frac{Z-z}{d},a_1]}(t)$$

$$+\sum_{n=1}^{k-1}\{[(A_n - d(t-a_n))]I_{[a_n,b_n]}(t) + [(B_n + (r-d)(t-b_n))I_{[b_n,a_{n+1}]}(t)$$

$$+[(A_k - d(t-a_k))I_{[a_k,b_k]}(t) + [(B_k + (r-d)(t-b_k))I_{[b_k,T_z]}(t)$$

where $I_{[a,b]}(t) = \begin{cases} 1 & if \quad t \in [a,b] \\ 0 & otherwise \end{cases}$

and $T_z = b_k + \frac{Z - B_k}{r-d}$ (12)

Proof: The main results follow from

Proposition 2. However, it remains to show that $T_z = b_k + \frac{Z - B_k}{r - d}$. In the case that $N(T_Z)=k$, the inventory will increase from B_n to Z during $[b_k, T_z]$ according to $J(t) = B_k + (\gamma - d)(t - b_k)$. Hence, we get $T_z = b_k + \frac{Z - B_k}{r - d}$ by solving $Z = B_k + (\gamma - d)(T_z - b_k)$.

q.e.d.

Proposition 4: Given $N(T_Z)=k$,

$$\begin{aligned}\int_0^{T_z}(C^+J^+(t)+C^-J^-(t))dt={}&C^+[\int_0^{\frac{Z-z}{d}}(Z-dt)dt+\int_{\frac{Z-z}{d}}^{a_1}(z+(r-d)(t-\frac{Z-z}{d}))dt]\\&+C^+\{\sum_{n=1}^{k-1}\int_{a_n}^{b_n}[A_n-d(t-a_n)]^+dt+\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^+dt\\&+\int_{a_k}^{b_k}[A_k-d(t-a_k)]^+dt\}+\int_{b_k}^{T_z}[B_k+(r-d)(t-b_k)]^+dt\}\\&+C^-\{\sum_{n=1}^{k-1}\int_{a_n}^{b_n}[A_n-d(t-a_n)]^-dt+\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^-dt\\&+\int_{a_k}^{b_k}[A_k-d(t-a_k)]^-dt\}+\int_{b_k}^{T_z}[B_k+(r-d)(t-b_k)]^-dt\}\end{aligned}$$

Proof: This follows simply from Proposition 2.

q.e.d.

Proposition 5:

$$\int_{a_n}^{b_n}[A_n-d(t-a_n)]^+dt=\begin{cases}(b_n-a_n)[A_n-\frac{d}{2}(b_n-a_n)] & \textit{if}\quad A_n>B_n\geq 0\\ \frac{1}{2}\cdot\frac{A_n{}^2}{d} & \textit{if}\quad A_n>0>B_n\end{cases}$$

$$\int_{a_n}^{b_n}[A_n-d(t-a_n)]^-dt=\begin{cases}\frac{d}{2}\cdot(b_n-a_n-\frac{A_n}{d})^2 & \textit{if}\quad A_n>0>B_n\\ -(b_n-a_n)[A_n-\frac{d}{2}(b_n-a_n)] & \textit{if}\quad 0\geq A_n>B_n\end{cases}$$

$$\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^+dt=\begin{cases}(a_{n+1}-b_n)[B_n+\frac{r-d}{2}(a_{n+1}-b_n)] & \textit{if}\quad A_{n+1}>B_n\geq 0\\ \frac{(r-d)}{2}\cdot(a_{n+1}-b_n+\frac{B_n}{r-d})^2 & \textit{if}\quad A_{n+1}>0>B_n\end{cases}$$

$$\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^-dt=\begin{cases}\frac{1}{2}\cdot\frac{B_n{}^2}{(r-d)} & \textit{if}\quad A_{n+1}>0>B_n\\ -(a_{n+1}-b_n)[B_n+\frac{1}{2}(r-d)(a_{n+1}-b_n)] & \textit{if}\quad 0>A_{n+1}>B_n\end{cases}$$

$$\int_{b_k}^{T_Z}[B_k+(r-d)(t-b_k)]^+dt=\begin{cases}\frac{1}{2}\cdot\frac{Z^2-B_k{}^2}{(r-d)} & \textit{if}\quad B_k>0\\ \frac{1}{2}\cdot\frac{Z^2}{(r-d)} & \textit{if}\quad 0>B_k\end{cases}\qquad \textit{where}\ T_Z=b_k+\frac{Z-B_k}{r-d}$$

$$\int_{b_k}^{T_Z}[B_k+(r-d)(t-b_k)]^-dt=\frac{1}{2}\cdot\frac{B_k{}^2}{(r-d)}\qquad \textit{if}\quad 0>B_k\qquad \textit{where}\ T_Z=b_k+\frac{Z-B_k}{r-d}$$

Proof: Since

$$\int_{a_n}^{b_n}[A_n-d(t-a_n)]^+dt=\begin{cases}\int_{a_n}^{b_n}[A_n-d(t-a_n)]dt & \textit{if}\quad A_n>B_n\geq 0\\ \int_{a_n}^{a_n+\frac{A_n}{d}}[A_n-d(t-a_n)]dt & \textit{if}\quad A_n>0>B_n\end{cases}$$

$$\int_{a_n}^{b_n}[A_n-d(t-a_n)]^-dt=\begin{cases}-\int_{a_n+\frac{A_n}{d}}^{b_n}[A_n-d(t-a_n)]dt & \textit{if}\quad A_n>0>B_n\\ -\int_{a_n}^{a_n}[A_n-d(t-a_n)]dt & \textit{if}\quad 0\geq A_n>B_n\end{cases}$$

$$\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^+dt=\begin{cases}\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]dt & \textit{if}\quad A_{n+1}>B_n\geq 0\\ \int_{b_n-\frac{B_n}{r-d}}^{a_{n+1}}[B_n+(r-d)(t-b_n)]dt & \textit{if}\quad A_{n+1}>0>B_n\end{cases}$$

$$\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]^-dt=\begin{cases}-\int_{b_n}^{b_n-\frac{B_n}{r-d}}[B_n+(r-d)(t-b_n)]dt & \textit{if}\quad A_{n+1}>0>B_n\\ -\int_{b_n}^{a_{n+1}}[B_n+(r-d)(t-b_n)]dt & \textit{if}\quad 0\geq A_{n+1}>B_n\end{cases}$$

$$\int_{b_k}^{T_Z}[B_k+(r-d)(t-b_k)]^+dt=\begin{cases}\int_{b_k}^{b_k+\frac{Z-B_k}{r-d}}[B_k+(r-d)(t-b_k)]dt & \textit{if}\quad B_k>0\\ \int_{b_k-\frac{B_k}{r-d}}^{b_k+\frac{Z-B_k}{r-d}}[B_k+(r-d)(t-b_k)]dt & \textit{if}\quad 0>B_k\end{cases}\qquad \textit{where}\ T_Z=b_k+\frac{Z-B_k}{r-d}$$

$$\int_{b_k}^{T_Z}[B_k+(r-d)(t-b_k)]^-dt=\int_{b_k}^{b_k-\frac{B_k}{r-d}}[B_k+(r-d)(t-b_k)]dt\quad \textit{if}\quad 0>B_k\quad \textit{where}\ T_Z=b_k+\frac{Z-B_k}{r-d}$$

and

$$\int[A_n-d(t-a_n)]dt=A_n\cdot t-\frac{d}{2}(t-a_n)^2$$

$$\int[B_n+(r-d)(t-b_n)]dt=B_n+\frac{1}{2}(r-d)(t-b_n)^2$$

q.e.d.

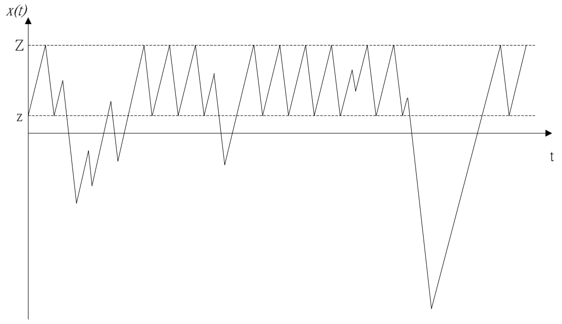


Figure 2: A sample path of $\{ J(t)\}_{t\geq 0}$ for a single unreliable machine

3. Simulation Method

The procedure for the proposed simulation can be described as follows;

First of all, we let Ω be the set of m random generated sample paths from $\{X_n ,Y_n\}_{n\geq 1}$, Ω_k the subset of those from Ω with exactly k failures during the first cycle $[0,\boldsymbol{T}_Z]$. For convenience, we let $m_k = |\Omega_k|$ (the cardinality of Ω_k), T_k^i be the cycle length for the ith sample from Ω_k and C_k^i the corresponding total cost during $[0, T_k^i]$. Then we get

$$P\{N(T_z)=k\} \approx P\{\Omega_k\} = \frac{m_k}{m}$$

$$E(T_z|N(T_z)=k) \approx \frac{1}{m_k}\sum_{i=1}^{m_k} T_k^i$$

$$E\int_0^{T_z}[C^+x^+(t)+C^-x^-(t)]dt|N(T_z)=k) \approx \frac{1}{m_k}\sum_{i=1}^{m_k} C_k^i$$

Hence, the simulation will be reduced to 1) check whether each sample from Ω is in Ω_k (See Proposition 6) for each k, and 2) calculate T_k^i and C_k^i for the ith sample in Ω_k (See Proposition 7,8).

Proposition 6: The sample $\{x_n ,y_n\}_{n\geq 1} \in \Omega_k$ iff it satisfies the following property:

$0 < z < A_1 < Z$, $A_2, A_3,\ldots, A_k < Z$ & $A_{k+1} \geq Z > 0$

Proof: The proof of such a case for $k = 2$ can be illustrated in Figure 3.

q.e.d.

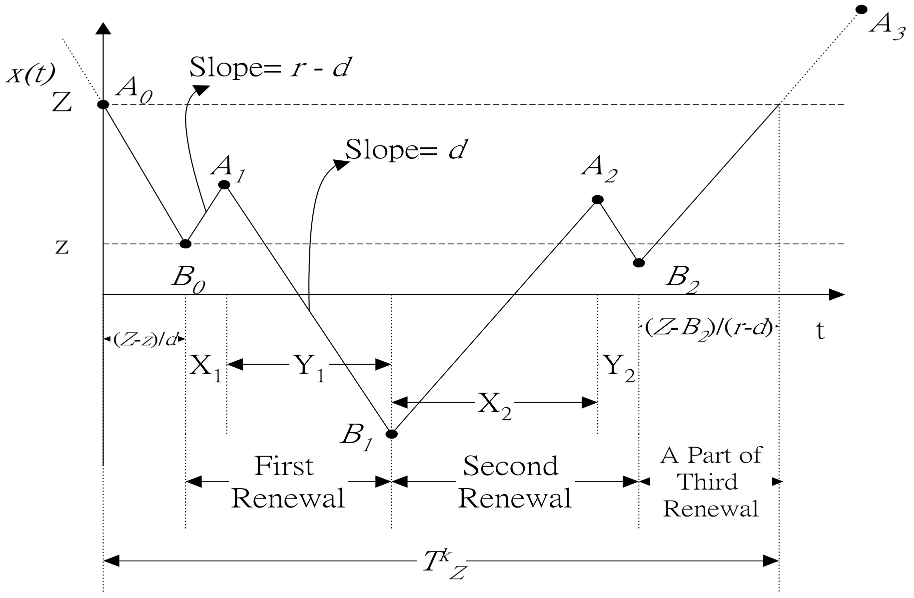


Figure 3 A sample path of $\{ J(t)\}_{t\geq 0}$ between successive Z's when $k = 2$.

Proposition 7: For each $\{x_n^{(i)}, y_n^{(i)}\}_{n=1,2,\dots,L} \in \Omega_k$, with $i=1,2,\dots, m_k$. Then

$$T_k^i = \frac{Z-z}{d} + \sum_{n=1}^{k}(x_n^{(i)} + y_n^{(i)}) + \frac{Z-B_k^i}{r-d}$$

Proof: First of all, $T_Z^i = b_k^i + \dfrac{Z-B_k^i}{r-d}$ (by proposition 1). The rest then follows from the fact that $b_k^i = \dfrac{Z-z}{d} + \sum_{n=1}^{k}(X_n^{(i)} + Y_n^{(i)})$ (by proposition 1).

q.e.d.

Proposition 8: For each $\{x_n^{(i)}, y_n^{(i)}\}_{n=1,2,\dots,L} \in \Omega_k$, with $i=1,2,\dots,m_k$. Then

$$\begin{aligned} C_k^i = C^+[\int_0^{\frac{Z-z}{d}} (Z-dt)dt + \int_{\frac{Z-z}{d}}^{a_1^i} (z+(r-d)(t-\frac{Z-z}{d}))dt] \\ + C^+\{\sum_{n=1}^{k-1}\int_{a_n^i}^{b_n^i}[A_n^i - d(t-a_n^i)]^+ dt + \int_{b_n^i}^{a_{n+1}^i}[B_n^i + (r-d)(t-b_n^i)]^+ dt \\ + \int_{a_k^i}^{b_k^i}[A_k^i - d(t-a_k^i)]^+ dt\} + \int_{b_k^i}^{T_k^i}[B_k^i + (r-d)(t-b_k^i)]^+ dt\} \\ + C^-\{\sum_{n=1}^{k-1}\int_{a_n^i}^{b_n^i}[A_n^i - d(t-a_n^i)]^- dt + \int_{b_n^i}^{a_{n+1}^i}[B_n^i + (r-d)(t-b_n^i)]^- dt \\ + \int_{a_k^i}^{b_k^i}[A_k^i - d(t-a_k^i)]^- dt\} + \int_{b_k^i}^{T_k^i}[B_k^i + (r-d)(t-b_k^i)]^- dt\} \end{aligned}$$

Proof: This follows from Proposition 4.

q.e.d.

Algorithm:

Step 0: Set $m_k = 0$ for each $k = 0,1,2,\dots$.

Step 1: Generate m random samples $\{ X_n^{(j)}, Y_n^{(j)} \}_{n=1,2,\dots,L}$ ($j = 1,2,\dots,m$).

Step 2: To each $k = 0,1,2,\dots$, check each sample $\{ X_n^{(j)}, Y_n^{(j)} \}_{n=1,2,\dots,L}$ by applying proposition 5 whether it belongs to Ω_k.

Step 3: If yes, calculate T_k^i and C_k^i

$$T_k^i = \frac{Z-z}{d} + \sum_{n=1}^{k}(X_n^{(j)} + Y_n^{(j)}) + \frac{Z-B_k^j}{r-d}$$

(by Proposition 7)

$$C_k^i = \int_0^{T_k^j}[C^+J^+(t) + C^-J^-(t)]dt$$

(by Proposition 8) and also set $m_k = m_k + 1$.

Step 4: After going through all samples in Ω, we reach the final m_k and thus

$$P\{N(T_z)=k\} \approx P\{\Omega_k\} = \frac{m_k}{m}$$

$$E(T_z | N(T_z)=k) \approx \frac{1}{m_k}\sum_{i=1}^{m_k} T_k^i$$

$$E\int_0^{T_z}[C^+x^+(t) + C^-x^-(t)]dt | N(T_z)=k) \approx \frac{1}{m_k}\sum_{i=1}^{m_k} C_k^i$$

Step 5: Calculate

$$H = \frac{1}{E(T_Z)} E[\int_0^{T_Z} C^+J^+(t) + C^+J^+(t))dt]$$

4. Summary and Further Study

A simulation method is proposed to first estimate the expected cost rate due to both inventory holding and backlog under the (z, Z) production control policy ($z < Z$), and then obtain the optimal (z, Z) values for a single unreliable machine with buffer. An algorithm is presented to illustrate how to estimate the inventory cost rates for various pairs of (z, Z) and how to get the minimal one among them.

Further study can be taken by further taking: (a) the preventive maintenance or (b) the product defective rate into consideration for a single unreliable machine with final buffer, or by concentrating on (c) an unreliable series production system with buffers.

References

1. Akella, R., Kumar, P.R. (1986). “Optimal control of production rate in a failure prone manufacturing system”. *IEEE Trans. Automat. Contr,. 31(2),* 116-126.
2. Bielecki, T., Kumar, P. R. (1988). “Optimality of zero-inventory policies for unreliable manufacturing systems”. *Operations Research*3, *6(4),* 532-541.
3. Bai, S. X., Gershwin, S. B. (1990). “Scheduling manufacturing systems with work-in-process inventory”. *Proceedings of the 29th Conference on Decision and*

Control. 557-564.

4. Bai, S. X., Gershwin, S.B. (1995). “Scheduling manufacturing systems with work-in-process inventory control: single-part-type systems”. *IIE Transactions, 27*, 599-617.
5. Ching, W.K., (1998). “Optimal (s, S) policies for manufacturing systems of unreliable machines in Tandem”. *1998 Proceedings Annual Reliability and Maintainability Symposium.*, 365-370.
6. Fong, N. T., Zhou, X. Y. (2000). “Optimal feedback controls in deterministic two-machine flowshops with finite buffers”. *IEEE Trans. Automat. Contr., 45(6),* 1198-1203.
7. Mbihi , J., Malhame, R.P. (1999). “A suboptimal class of decentralized hedging production policies in single part-type, stochastic m-machine flow shop”. *Proceedings of the 38th Conference on Decision and Control*, 2282-2287.
8. Ross, S. M. (1993). *Introduction to Probability Models*, 5th eds. USA: Academic Press.
9. Sharifnia, A. (1988). “Production control of a manufacturing system with multiple machine states”. *IEEE Trans. Automat. Contr., 33(7)*, 620-625.