Bit Error Period Determination and Optimal Frame Length Prediction for a Noisy Communication Channel

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Abstract

The investigation presents an evaluation of data exchange efficiency for a packet switched “point to point” communication channel in a noisy environment. This paper shows the influence of the frame length on the communication channel performance and presents a stage of a research for determining the error rate during data exchange. The results obtained make it possible to reach a maximum communication channel throughput by dynamic and adaptive choice of the frame length in the sending system, according to the current external conditions.

Keywords: Telecommunications and networking, communication channel performance, data exchange in noisy conditions.

Introduction

In packet switched networks the data segments are transmitted as independent information entities via the communication channel. The presence of high noisy environment results to a high frequency of communication errors. This causes the loss of a significant quantity of data frames in the receiving system. The restoration of lost data is done by retransmission. In this case, the performance of the communication channel depends strongly on the length of the exchanged data frames (see Ci and Sharif 2002; Borovska and Mohsen 2004; Elliott 1965). The goal of the presented analysis is a creation of:

• a simple analytic model on the base of which the relation between communication channel efficiency, frame length and bit error period to be found;
• a way to determine the value of the bit error period by the sending system;
• a solution for selection of suitable prediction bit error period value by the sending system in cases when the calculation of this value is impossible.

The obtained results make it possible to integrate adaptive algorithm for optimal frame length control in the sending system and to reach a maximum communication channel throughput.

Concept

The investigation is based on the following assumptions:

• communication errors are characterized by a periodic distribution;
• validity control of the exchanged information is made by error-detecting codes, so, even an isolated single-bit error leads to the damage and the rejection of the whole data frame.

The general frame format for a data packet is illustrated in Fig. 1. The header is a control data field of fixed length. Data is a variable length payload field that contains a segment of user data.

Fig. 1. General frame format for a data packet.
$L$ is the total length of the frame, $d$ is the frame payload size and $c$ is the header size of the transmitting frame, including of MAC and PHY layers overheads.

The payload size is selected by the transmitting system. In this case, the optimal choice should be found among the following two opposite trends:

- for an error-free communication channel the increase of user data length promotes the channel efficiency due to increasing the ratio of the payload size to the total frame length;
- for a high-error rate communication channel the increase of user data length decreases the channel efficiency, as a result of erroneous frames retransmission.

**Analytic Model**

A time diagram of data exchange for maximum loaded communication channel with bit errors of periodical nature is illustrated by Fig. 2. The efficiency factor of the communication channel can be expressed by the following equation:

$$K_{\text{eff}} = \frac{D}{N},$$  \hspace{1cm} (1)

where: $D$ [bits] is the total quantity of correctly received data; $N$ [bits] is the total quantity of transmitted data.

The total number of transmitted frames is presented by:

$$\text{Transmitted Frames} = \frac{N}{c+d}. \hspace{1cm} (2)$$

The total number of correctly received frames is presented by:

$$\text{Received Frames} = \frac{D}{d}. \hspace{1cm} (3)$$

The total number of lost frames is presented by:

$$\text{Lost Frames} = \frac{N}{T_{\text{error}}} \hspace{1cm} (4)$$

where $T_{\text{error}}$ [bits] is the bit error period.

The following equation is a result of Eqs. (2), (3) and (4):

$$\frac{N}{T_{\text{error}}} + D = \frac{N}{d} - \frac{N}{c+d}. \hspace{1cm} (5)$$

Eqs. (6) and (7) are a result of transforming the equation (5):

$$K_{\text{eff}} = \left(1 - \frac{1}{1 + \frac{c+d}{d} \frac{1}{T_{\text{error}}}}\right). \hspace{1cm} (6)$$

$$K_{\text{eff}} = \frac{d}{c+d} - \frac{1}{T_{\text{error}}}. \hspace{1cm} (7)$$

The following equation is obtained by replacing (1) in (7):

$$K_{\text{eff}} = \frac{d}{c+d} - \frac{1}{T_{\text{error}}}. \hspace{1cm} (8)$$

The result of plotting Eq. (8) for a variety of frame payload sizes and of bit error periods is shown in Fig. 3. This is an example of data exchange via the PPP (Point to Point Protocol) communication protocol (see Simpson 1994), under the following conditions:

Fig. 2. Data exchange for maximal loaded communication channel.
control data length in the frame is \( c = 10 \) bytes;

- minimal user data length in the frame is \( d_{\text{min}} = 2 \) bytes, maximal user data length is \( d_{\text{max}} = 1,024 \) bytes.

Practically, only the values for \( T_{\text{error}} > d + c \) should be considered (see Naydenov and Stoyanov 2005).

Similar results have been obtained by Lettieri and Srivastava (1998).

This plot shows that for each channel environmental condition in terms of bit error rate it is possible to choose a certain optimum value for the frame payload size for which \( K_{\text{eff}} \) reaches the highest value. This value can be expressed as follows:

\[
K_{\text{eff}} = \left( \frac{d}{c+d} \right)^2 \left( \frac{1}{T_{\text{error}}} \right),
\]

(9)

\[
K'_{\text{eff}} = \frac{c+d-d}{(c+d)^2} - \frac{1}{T_{\text{error}}},
\]

(10)

\[
\frac{c}{(c+d)^2} - \frac{1}{T_{\text{error}}} = 0,
\]

(11)

\[
c*T_{\text{error}} - (c+d)^2 = 0,
\]

(12)

\[
d^2 + 2c*d + c^2 - c*T_{\text{error}} = 0.
\]

(13)

Equations (14, 15) are the result of solving equation (13) and allow the calculation of optimal value for frame payload size or for frame size, depending on channel environment conditions in terms of bit error period:

\[
d = -c + \sqrt{c*T_{\text{error}}},
\]

(14)

\[
L_{\text{opt}} = \sqrt{c*T_{\text{error}}}. \]

(15)

**Bit Error Period Determination**

The analysis of the presented model imposes the conclusion that the prediction of the optimal frame length is reduced mainly to determination of the bit error period value in the communication channel. In order to do this the transmitting system regularly determines the count of erroneous frames \( F_{\text{err}} \) for a fixed time period called prediction interval. The period of the communication error in the prediction interval can be calculated according to this formula:

\[
T_{\text{error}} = \frac{N}{F_{\text{err}}} * L \ [\text{bit}],
\]

(16)

where: \( L \) is the length of the transmitted frames; \( N \) is the total count of transmitted frames in the prediction interval; \( F_{\text{err}} \) is the count of erroneous frames in the prediction interval.

Eq. (16) allows determination with adequate accuracy of the packet error period \( T_{\text{error}} \) only in cases when its actual value exceeds the frame length, \( T_{\text{error}} > L \), i.e., when not all transmitted in the prediction interval frames have been erroneous.

Calculation of the bit error period value according to Eq. (16) in case that all transmitted frames in the prediction interval are erroneous would always result to \( T_{\text{error}} = L \) because \( F_{\text{err}} = N \). This result only reflects one variant of many possible. In reality, the bit error period could have random values \( L_{\text{min}} < T_{\text{error}} \leq L \), because every value in this range would lead to errors in all frames. In this case, the bit error period can be defined as incompletely determined, and the interval \( L_{\text{min}} \div L \) as critical, because there are no real criteria to calculate the exact value of \( T_{\text{error}} \) in the sending system. The solution is to select a proper prediction value among the many possible in the critical interval, which would lead to possibly highest expected communication channel throughput.

![Fig. 3. \( K_{\text{eff}} \) dependence on the frame payload size \( d \) and bit error period in accordance with Eq. (8).](image_url)
The selected value for the bit error period influences the channel throughput by the respective change of the frame length. Obviously, the maximum efficiency will be reached only if the chosen prediction value matches the actual one. In every other case, the channel throughput will be to some extent smaller than the maximum possible because the selected frame length will no longer be optimal in respect of the actual conditions. This means that the expected efficiency of communication channel’s usage at chosen prediction value \( j \), should be assessed integrally for all possible actual values of the bit error period in the critical interval \( L_{\text{min}} \div L \). Quantitative expression of this assessment accounts for the factor of expected efficiency, presented by the following formula:

\[
K_{\text{eff,pr}} = \sum_{i=L_{\text{min}}}^{L} p_i \cdot K_{\text{eff,ij}}
\]

where: \( K_{\text{eff,pr}} \) is the factor of expected efficiency at chosen prediction value of the bit error period equal to \( j \); \( p_i \) is the probability that the actual bit error period is equal to \( i \); \( K_{\text{eff,ij}} \) is the efficiency factor of communication channel throughput at actual bit error period equal to \( i \) and frame length equal to \( L_j \); and \( L_j \) is the optimal frame length at bit error period equal to \( j \).

If the actual bit error period can accept all values from the critical interval with equal probability, Eq. (3) can be presented as follows:

\[
K_{\text{eff,pr}} = \sum_{i=L_{\text{min}}}^{L} \frac{1}{L-L_{\text{min}}+1} \cdot K_{\text{eff,ij}}
\]

The selection of prediction value in similar situation will be illustrated with an example, in which the transmitting system has to determine the value of the bit error period \( T_{\text{error}} \), using the following conditions:

- data exchange is made with communication protocol PPP (Point to Point Protocol);
- control data length in the frame is \( c = 10 \) bytes;
- minimal user data length in the frame is \( d_{\text{min}} = 2 \) bytes, maximal user data length is \( d_{\text{max}} = 1,024 \) bytes;
- the frame length during the prediction interval was \( L_{\text{old}} = 130 \) bytes;
- all transmitted frames in this interval have been hit.

The result is a case which is incompletely determined. Hence a prediction value of the bit error period has to be selected from the critical interval \( C_{\text{int}} = 12 \div 130 \) bytes. For this purpose, it is necessary to evaluate the expected average efficiency of communication channel throughput for every value of \( T_{\text{error}} \in C_{\text{int}} \) according to Eq. (17). For clear presentation of the analysis results the following assumptions will be made:

- possible real values of the bit error period in the critical interval are four, respectively 120, 90, 60 and 30 bytes;
- the probabilities of their occurrence are equal;
- the expected average efficiency factor is calculated with accuracy of up to four decimal characters.

Graphics reflecting the influence of the user data length in the frame on the coefficient of communication channel efficiency \( K_{\text{eff}} \) for the selected four values of the bit error period are shown on Figs. 4, 5, 6 and 7. The results are received via simulating data exchange with the communication protocol PPP (Point to Point Protocol).

Fig. 4 presents the case of evaluation of the expected average communication channel efficiency at prediction bit error period value \( T_{\text{error}} = 120 \) bytes.

The resulting optimal user data length in the frame \( d_{\text{opt}} = 25 \) bytes in position \( A \) results to a communication channel efficiency factor \( K_{\text{eff,A}} = 0.5059 \). Positions \( B \), \( C \) and \( D \) reflect the cases in which the actual period of the bit error differs from the selected prediction value.

The calculation of the expected average efficiency according to Eqs. (17) and (18) is narrowed down to Eqs. (19) and (20):

\[
K_{\text{eff,pr}} = p_A K_{\text{eff,A}} + p_B K_{\text{eff,B}} + p_C K_{\text{eff,C}} + p_D K_{\text{eff,D}},
\]

(19)

\[
K_{\text{eff,pr}} = 0.25 \cdot (K_{\text{eff,A}} + K_{\text{eff,B}} + K_{\text{eff,C}} + K_{\text{eff,D}}),
\]

(20)
where: $p_X$ is the probability for appearance of a packet error with a period corresponding to the curve to which position $X$ belongs (for $X = A, B, C, D$); $K_{\text{eff}_X}$ is the value of the efficiency factor in position $X$. The results of the calculations are summarized in Table 1.

Fig. 5 presents the case of evaluation of the expected average communication channel throughput efficiency at prediction bit error period value $T_{\text{error}} = 90$ bytes. The results of the calculations are summarized in Table 2.

Fig. 6 presents the case of evaluation of the expected average communication channel throughput efficiency at prediction bit error period value $T_{\text{error}} = 60$ bytes. The results of the calculations are summarized in Table 3.

Fig. 7 presents the case of evaluation of the expected average communication channel throughput efficiency at prediction bit error period value $T_{\text{error}} = 30$ bytes. The results of the calculations are summarized in Table 4.

The results for the four analyzed cases are summarized in Table 5. Maximum is the value of the expected average efficiency factor $K_{\text{eff}_\text{pr} 60}$.

Table 1. Expected average efficiency for $T_{\text{error}} = 120$ bytes and $d_{\text{opt}} = 25$ bytes.

<table>
<thead>
<tr>
<th>$K_{\text{eff}_A}$</th>
<th>$K_{\text{eff}_B}$</th>
<th>$K_{\text{eff}_C}$</th>
<th>$K_{\text{eff}_D}$</th>
<th>$K_{\text{eff}_{\text{pr} 120}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5059</td>
<td>0.4365</td>
<td>0.2976</td>
<td>0.0000</td>
<td>0.3100</td>
</tr>
</tbody>
</table>

Table 2. Expected average efficiency for $T_{\text{error}} = 90$ bytes and $d_{\text{opt}} = 20$ bytes.

<table>
<thead>
<tr>
<th>$K_{\text{eff}_A}$</th>
<th>$K_{\text{eff}_B}$</th>
<th>$K_{\text{eff}_C}$</th>
<th>$K_{\text{eff}_D}$</th>
<th>$K_{\text{eff}_{\text{pr} 90}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000</td>
<td>0.4444</td>
<td>0.3333</td>
<td>0.0000</td>
<td>0.3194</td>
</tr>
</tbody>
</table>

Table 3. Expected average efficiency for $T_{\text{error}} = 60$ bytes and $d_{\text{opt}} = 15$ bytes.

<table>
<thead>
<tr>
<th>$K_{\text{eff}_A}$</th>
<th>$K_{\text{eff}_B}$</th>
<th>$K_{\text{eff}_C}$</th>
<th>$K_{\text{eff}_D}$</th>
<th>$K_{\text{eff}_{\text{pr} 60}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4750</td>
<td>0.4333</td>
<td>0.3500</td>
<td>0.1000</td>
<td>0.3396</td>
</tr>
</tbody>
</table>
This case determines the selection of prediction bit error period value $T_{\text{error}} = 60$ bytes, leading to selection of optimal user data length in the frame $d_{\text{opt}} = 15$ bytes, see Fig. 6 and Table 3.

While accomplishing such analysis it must be taken in mind that the real communications channel throughput efficiency will always differ from the calculated expected average efficiency, because the factor of expected average efficiency is a generalized virtual dimension. The expected average efficiency for a given frame length is calculated versus all possible values of the bit error period in the critical interval, while the real efficiency is calculated only towards the real bit error period value. That’s why the quantitative difference between the expected average communication channel throughput efficiency and the real channel throughput efficiency will depend on the difference between the selected prediction bit error period and its real value.

The presented differences, for selected prediction value of the bit error period $T_{\text{error}} = 60$ bytes, are shown in Table 6.

**Table 5. A summary of expected average efficiency results.**

<table>
<thead>
<tr>
<th>Critical Interval $C_{\text{opt}} = 12 \div 130$ bytes</th>
<th>Possible equally probable values for $T_{\text{error}} \rightarrow 30$, 60, 90, and 120 bytes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction value of $T_{\text{error}}$</td>
<td>Expected average efficiency</td>
</tr>
<tr>
<td>$T_{\text{error}} = 30$ bytes</td>
<td>$K_{\text{eff}}_{\text{pr} \ 30} = 0.2901$</td>
</tr>
<tr>
<td>$T_{\text{error}} = 60$ bytes</td>
<td>$K_{\text{eff}}_{\text{pr} \ 60} = 0.3396$</td>
</tr>
<tr>
<td>$T_{\text{error}} = 90$ bytes</td>
<td>$K_{\text{eff}}_{\text{pr} \ 90} = 0.3194$</td>
</tr>
<tr>
<td>$T_{\text{error}} = 120$ bytes</td>
<td>$K_{\text{eff}}_{\text{pr} \ 120} = 0.3100$</td>
</tr>
</tbody>
</table>

**Table 6. Differences for selected prediction value of the bit error period $T_{\text{error}} = 60$ bytes.**

<table>
<thead>
<tr>
<th>Critical Interval $C_{\text{opt}} = 12 \div 130$ bytes</th>
<th>Selected prediction value of $T_{\text{error}}$</th>
<th>Expected average efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{error}}$</td>
<td>$K_{\text{eff}}_{\text{pr} \ 60}$</td>
<td>Real efficiency</td>
</tr>
<tr>
<td>60 bytes</td>
<td>0.3396</td>
<td>$K_{\text{eff}}_{A} = 0.4750$</td>
</tr>
<tr>
<td>30 bytes</td>
<td>0.1000</td>
<td>$K_{\text{eff}}_{B} = 0.4333$</td>
</tr>
<tr>
<td>60 bytes</td>
<td>0.3500</td>
<td>$K_{\text{eff}}_{C} = 0.4333$</td>
</tr>
<tr>
<td>90 bytes</td>
<td>0.3500</td>
<td>$K_{\text{eff}}_{D} = 0.4750$</td>
</tr>
<tr>
<td>120 bytes</td>
<td>0.3100</td>
<td></td>
</tr>
</tbody>
</table>

**Optimal Frame Length Prediction**

This can be done in accordance with the following steps:
- the sending system determines the number of erroneous frames in a fixed time period, named prediction period;
- if the erroneous frame count is less than the total transmitted frames count, the bit error period value $T_{\text{error}}$ is calculated according to Eq. (16);
- otherwise, a prediction value of the bit error period $T_{\text{error}}$ is determined, according to the following order:
  - the expected efficiency factor is calculated for every value in the critical interval according to Eq. (17);
  - the value with maximum expected efficiency factor is chosen for a prediction value;
- the optimal frame length $L_{\text{opt}}$ is calculated according to Eq. (15).

This algorithm for optimal frame length prediction in the sending system is presented in Fig. 8, where: $L_{\text{old}}$ is the length of the transmitted frames in the prediction interval; $N$ is the total count of transmitted frames in the

![Fig. 7. Evaluation of the prospective average efficiency at prediction value $T_{\text{error}} = 30$ bytes.](image)

**Table 4. Expected average efficiency for $T_{\text{error}} = 30$ bytes and $d_{\text{opt}} = 7$ bytes.**

<table>
<thead>
<tr>
<th>$K_{\text{eff}}_{A}$</th>
<th>$K_{\text{eff}}_{B}$</th>
<th>$K_{\text{eff}}_{C}$</th>
<th>$K_{\text{eff}}_{D}$</th>
<th>$K_{\text{eff}}_{\text{pr} \ 30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3534</td>
<td>0.3339</td>
<td>0.2950</td>
<td>0.1784</td>
<td>0.2901</td>
</tr>
</tbody>
</table>

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Fig. 8. Algorithm for optimal frame length prediction in the sending system.

Results presented in Table 5 show the existence of a dependency between the selected prediction bit error period value $T_{\text{error}}$ and the factor of expected average efficiency $K_{\text{eff,pr}}$. With equal probabilities for occurrence of the possible bit error period values in the critical interval, with increasing the value of $T_{\text{error}}$ at first $K_{\text{eff,pr}}$ increases too, and then it starts to decrease. Therefore an optimal prediction value exists at which the expected average communications channel throughput efficiency is maximum. This can be confirmed in the forthcoming analysis of the expected average efficiency for a bigger set of possible values of the bit error period in the range of the critical interval and even distribution of the probabilities for their occurrence.

**Conclusion and Future Work**

This paper presents the results of an analysis showing the influence of the frame length on the communication channel performance in noisy environment conditions. A simple analytic model is created. The relation between the efficiency factor and both the frame payload size and the bit error period is defined and analyzed. An equation for calculating of the frame payload size optimal value for a given channel bit error rate is deduced.

As a final result a solution for bit error period value recognition and an adaptive frame length selection is presented. The purpose of this research is to reach maximum efficiency of data exchange. On the base of the presented results an adaptive algorithm could be incorporated into the transmitting system aiming maximum effective use of the communication channel.

**References**


