Intervalized Similarity and Star Products

Pratit Santiprabhob
Faculty of Science and Technology, Assumption University
Bangkok, Thailand

Abstract

The author and his colleagues have previously defined the Similarity and Star fuzzy relational products. The products have been developed as a means to tackle certain real-world problems for which the semantics of the classical fuzzy relational products has proven somewhat insufficient. The author into a family of products has later extended the two products. This family of products is further extended in this paper to accommodate interval membership degrees. This consequently leads to the added ability of the products to process information whose uncertainty is more suitably expressed in terms of interval membership degrees instead of a single point/value membership degree.

Keywords: Fuzzy relation, fuzzy relational product, similarity product, star product, interval computation, qualitative information processing.

Introduction

Fuzzy relation, which is basically a fuzzy set with a multidimensional universe of discourse, has long been used to capture and represent qualitative relationships between sets of real-world objects. The so-called fuzzy relation is mostly employed in the form of fuzzy two-dimensional relations or alternatively called fuzzy binary relation. This fuzzy relation together with a group of fuzzy relational products has been shown to be applicable to a variety of application areas that require qualitative information processing. In such application areas, it is typically needed to deal with inherent uncertainty or imprecision. Fuzzy relation and its accompanying fuzzy relational products have become a means by which it can make machines, i.e. computers, possess a part of human intelligence in qualitative processing of imprecise, but meaningful information.

Exemplary application areas include information retrieval (Kohout et al. 1984; Kohout and Bandler 1985; Jiamthapthaksin and Santiprabhob 2000), medical diagnosis (Bandler and Kohout 1981; Kohout and Bandler 1990), information protection (Santiprabhob and Kohout 1992; Santiprabhob and Kohout 1993; Kohout and Santiprabhob 2000), and social network analysis (Dowpiset and Santiprabhob 1998). In such applications, the data from each respective application domains, which are to be processed, are represented in terms of fuzzy relations. Depending on the semantics of the eventual results, appropriate fuzzy relational products are applied to different combinations of the relations.

The classical fuzzy relational products include the Circle product (sometimes called the fuzzy compositional operator), and the family of BK products introduced by Kohout et al. (1984); Kohout and Bandler (1985); Bandler and Kohout (1980); Kohout and Bandler (1990); Bandler and Kohout (1981); Bandler and Kohout (1987). These products include Triangle Subproduct, Triangle Superproduct and Square product. In the course of our research into new application domains, we have found that the existing classical products mentioned do not always
provide us with the desired semantics. This has led to our development of a new family of fuzzy relational products (Santiprabhob 1998), namely *Similarity product* and *Star product*. A comprehensive review of the classical products, as well as that of the two newly introduced products can be found in (Santoprabhob et al. 2000).

These two new products have subsequently been extended into a few more variations by the author in (Santiprabhob 2001). The two products together with their variations provide an alternative semantics that enhances our ability in qualitatively processing of information represented by fuzzy relations.

Up to now the products have only been defined to operate on fuzzy relations with single point/value membership degrees. Further investigation into certain application domains has yet to show that it may be more desirable in some circumstances to represent and manipulate information by means of fuzzy relations with interval membership degrees. Hence, there is a need to extend the products for these interval membership fuzzy relations. This paper, first, reviews the family of *Similarity* and *Star* products. Subsequently, the extensions of the products to accommodate fuzzy relations with interval membership degrees are proposed. In the end, an example suggesting how these newly proposed products may be used is given as a guideline.

**Fuzzy Relations**

Fuzzy relations discussed in this paper are assumed to be fuzzy binary relations. Making note that this can be assumed without a loss of generality, since a fuzzy relation of higher dimensions can always be represented as multiple two-dimensional fuzzy relations. A fuzzy relation with single point/value membership degrees has its membership degrees as any real numbers in the interval [0, 1]. On the other hand, a fuzzy relation with interval membership degrees has its membership degrees as intervals of real numbers, which are subsets of the interval [0, 1]. These can be defined mathematically as follows:

For a single point/value membership fuzzy relation $R$, each of its membership degrees can be represented as

$$\mu_R(x, y) \in [0,1].$$

On the other hand, for an interval membership fuzzy relation $R'$, each of its membership degrees can be represented as

$$\mu_{R'}(x, y) = [a, b] \subseteq [0,1].$$

Examples of fuzzy relations with single point / value membership degrees $R$ and $S$ are shown in Figs. 1 and 2, while those of fuzzy relations with interval membership degrees $R'$ and $S'$ are shown in Figs. 3 and 4. These fuzzy relations are used to facilitate the discussion of our fuzzy relational products in subsequent Sections.

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$ 0.6</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>$x_2$ 0.0</td>
<td>0.02</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Fig. 1. Fuzzy relation $R$

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$ 0.0</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$z_2$ 0.1</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>$z_3$ 0.0</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 2. Fuzzy relation $S$

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$ [0.4,0.5]</td>
<td>[0.7,0.9]</td>
<td>[0.1,0.2]</td>
</tr>
<tr>
<td>$x_2$ [0.0,0.3]</td>
<td>[0.1,0.5]</td>
<td>[0.0,0.4]</td>
</tr>
</tbody>
</table>

Fig. 3. Fuzzy relation $R'$

<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$ [0.0,0.5]</td>
<td>[0.4,0.5]</td>
<td>[0.3,0.6]</td>
</tr>
<tr>
<td>$z_2$ [0.0,0.5]</td>
<td>[0.7,0.9]</td>
<td>[0.8,0.9]</td>
</tr>
<tr>
<td>$z_3$ [0.0,0.5]</td>
<td>[0.1,0.2]</td>
<td>[0.0,0.2]</td>
</tr>
</tbody>
</table>

Fig. 4. Fuzzy relation $S'$

The two example fuzzy relations in each respective version can, in practice, represent
some kind of relationship between pairs of objects \((x', z')\) and \((z', y')\) for which there is a common set of objects \((z')\) in both relations. For example, the relation \(R\) (or \(R'\)) could represent the relationships between individual tourists \((x')\) and (desired) attraction features \((z')\). While the relation \(S\) (or \(S'\)) may represent the relationships between attraction features (characterizing the attractions) \((z')\) and the attractions \((y')\). Then, different fuzzy relational products can be applied to the two relations to derive the resulting relationships between the tourists \((x')\) and the attractions \((y')\) according to their desired attraction features with different semantics.

**Similarity Product and Its Variations**

The semantics of Similarity product has proven essential to the use of fuzzy relations in Social Network Analysis (Dowpis and Santiprabhob 1998). In such analysis, the similarities among opinions/comments expressed by individuals in a given society are of concern. Such similarities cannot be readily determined by the application of the classical products. This has led to our development of the Similarity product as defined below.

**Similarity product** is defined in terms of membership degree calculation as:

\[
\mu_{R \land S}(x_i, y_k) = \frac{\sum_j (1 - |\mu_R(x_j, z_j) - \mu_S(z_j, y_k)|)}{\sum_j}
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the degree in which one object among objects \(z's\) relates to the object \(x\) in \(R\), and is similar to the counterpart that relates to the object \(y\) in \(S\).

In the definition of the Similarity product above, the absolute difference in their relationships to a common object \(z\) between an object \(x\) in \(R\) and an object \(y\) in \(S\) is defined as:

\[
|\mu_R(x_i, z_j) - \mu_S(z_j, y_k)|
\]

Therefore, the degree of similarity between the two objects \(x\) and \(y\) with respect to their relationships to a particular object \(z\) can be defined as:

\[
1 - |\mu_R(x_i, z_j) - \mu_S(z_j, y_k)|
\]

The Similarity product, then, averages out the degrees of similarity over all the common objects \(z's\) for each pair of objects \((x, y)\). The result of the application of this Similarity product to the example fuzzy relations of Figs. 1 and 2 is shown in Fig. 5.

\[\begin{array}{ccc}
  y_1 & y_2 & y_3 \\
  x_1 & 0.50 & 0.90 & 0.90 \\
  x_2 & 0.97 & 0.57 & 0.43 \\
\end{array}\]

Fig. 5. Resulting fuzzy relation \(R \land S\)

In addition to the original definition of the Similarity product described above, the average operator can be replaced by the max or min operator, and have the following variations of the Similarity product (Santiprabhob 2001).

**Similarity-Max product** is defined in terms of membership degree calculation as

\[
\mu_{R \lor S}(x_i, y_k) = \bigvee_j (1 - |\mu_R(x_j, z_j) - \mu_S(z_j, y_k)|)
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the maximum degree in which one object among objects \(z's\) relates to the object \(x\) in \(R\), and is similar to the counterpart that relates to the object \(y\) in \(S\).

**Similarity-Min product** is, on the other hand, defined in terms of membership degree calculation as:

\[
\mu_{R \land S}(x_i, y_k) = \bigwedge_j (1 - |\mu_R(x_j, z_j) - \mu_S(z_j, y_k)|)
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the minimum degree in which one object among objects \(z's\) relates to the object \(x\) in \(R\), and is similar to the counterpart that relates to the object \(y\) in \(S\).

The results of the application of the Similarity-Max product and Similarity-Min
product to the same example fuzzy relations are shown in Figs. 6 and 7, respectively.

\[
\begin{array}{ccc}
y_1 & y_2 & y_3 \\
x_1 & 0.80 & 0.90 & 1.00 \\
x_2 & 1.00 & 0.70 & 0.50 \\
\end{array}
\]

Fig. 6. Resulting fuzzy relation \( R \circ S \)

\[
\begin{array}{ccc}
y_1 & y_2 & y_3 \\
x_1 & 0.30 & 0.90 & 0.70 \\
x_2 & 0.90 & 0.50 & 0.40 \\
\end{array}
\]

Fig. 7. Resulting fuzzy relation \( R \circ S \)

**Star Product and Its Variations**

Notice that in analyzing the similarities, the strengths of (positive) correlation among the comments/ideas are very much important. A high degree of similarity between two people, saying nothing at all is not as indicative as a high degree of similarity between two people saying something (but not everything) in common. This then leads to the definition of the Star product.

**Star product** is defined in terms of membership degree calculation as:

\[
\mu_{R \circ S}(x, y) = \frac{\sum_j (\mu_R(x, z_j) \mu_S(z_j, y_k))}{\sum_j}
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the combined strength of their relationships to a common set of objects \(z\)’s.

In the definition of the Star product above, the combined strength of relationships of an object \(x\) in \(R\) and of an object \(y\) in \(S\), with respect to a common object \(z\) is defined as:

\[
(\mu_R(x, z_j) \mu_S(z_j, y_k))
\]

Like the Similarity product, the Star product, then averages out the combined relationship degrees over all the common objects \(z\)’s for each pair of objects \((x, y)\). The application of the Star product to the same example relations yields the result as shown in Fig. 8.

\[
\begin{array}{ccc}
y_1 & y_2 & y_3 \\
x_1 & 0.03 & 0.31 & 0.37 \\
x_2 & 0.01 & 0.05 & 0.05 \\
\end{array}
\]

Fig. 8. Resulting fuzzy relation \( R \ast S \)

Similar to the case of the Similarity product, the Star product can also be extended by replacing the average operator by the max or min operator. This leads to the following variations of the Star product (Santiprabhob 1991).

**Star-Max product** is defined in terms of membership degree calculation as:

\[
\mu_{R \circ S}(x, y) = \vee_j (\mu_R(x, z_j) \mu_S(z_j, y_k))
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the maximum strength of their relationships with respect to one of their common set of objects \(z\)’s.

**Star-Min product** is also similarly defined in terms of membership degree calculation as:

\[
\mu_{R \circ S}(x, y) = \wedge_j (\mu_R(x, z_j) \mu_S(z_j, y_k))
\]

This gives a degree of relationship for each pair of objects \((x, y)\) according to the minimum strength of their relationships with respect to one of their common set of objects \(z\)’s.

The results of the applications of the Max and Min variations of the Star Product are shown in Figs. 9 and 10, respectively.

\[
\begin{array}{ccc}
y_1 & y_2 & y_3 \\
x_1 & 0.08 & 0.56 & 0.64 \\
x_2 & 0.02 & 0.14 & 0.16 \\
\end{array}
\]

Fig. 9. Resulting fuzzy relation \( R \ast \circ S \)
Fig. 10. Resulting fuzzy relation $R \ast \wedge S$

**Intervalized Similarity Products**

In this Section, the fuzzy Similarity product and its variations previously described in Sections 3, are extended to fuzzy relations with interval membership degrees. These extended productions take the following forms.

*Similarity product* for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

$$\mu'_{R \circ S'}(x_i, y_k) = \frac{\sum_j ([1,1] - |\mu'_{R_i}(x_i, z_j) - \mu'_{S_i}(z_j, y_k)|)}{\sum_j}$$

The trick here is on how to define the absolute difference between two interval membership degrees. First, defining the two intervals representing interval membership degrees.

$$\mu'_{R_i}(x_i, z_j) = [l'_{R_i}(x_i, z_j), h'_{R_i}(x_i, z_j)]$$

$$\mu'_{S_i}(z_j, y_k) = [l'_{S_i}(z_j, y_k), h'_{S_i}(z_j, y_k)]$$

The normal subtraction of the two intervals can then be derived as follows:

$$\mu'_{R_i}(x_i, z_j) - \mu'_{S_i}(z_j, y_k) = [l'_{R_i}(x_i, z_j) - h'_{S_i}(z_j, y_k), h'_{R_i}(x_i, z_j) - l'_{S_i}(z_j, y_k)]$$

If the difference is interpreted as the interval subtraction shown above, it can easily be observed that the difference of two exactly equal intervals does not necessarily yield [0, 0]. Based on the semantics of the original Similarity product, the difference is more appropriately interpreted as a sort of Euclidean distance rather than the interval subtraction. Consequently, the absolute difference of the two intervals should instead be defined as follows:

$$|\mu_{R_i}(x_i, z_j) - \mu_{S_i}(z_j, y_k)| =$$

$$[\min(|l_{R_i}(x_i, z_j) - l_{S_i}(z_j, y_k)|,|h_{R_i}(x_i, z_j) - h_{S_i}(z_j, y_k)|),$$

$$\max(|l_{R_i}(x_i, z_j) - l_{S_i}(z_j, y_k)|,|h_{R_i}(x_i, z_j) - h_{S_i}(z_j, y_k)|)]$$

Then, the similarity can be computed using the normal interval subtraction since this is a subtraction of the resulting difference measure whose value is a subset of an interval [0, 1] from a fixed reference point [1, 1]. At the end, an average can be taken over the objects $z$’s as in the original definition by following the normal interval addition and the division of interval by real number.

As for the two variations of this Similarity product, namely the Similarity-Max product and the Similarity-Min product, in place of the average operation, will be needed to define the semantics of maximum and minimum operators on intervals of membership degrees. This must be done in the context of Similarity product. Note that the intention of these products is to determine the maximum or the minimum degree of similarity between the objects $x$ and $y$ with respect to individual objects $z$’s. The definitions of the products are hence given below.

*Similarity-Max product* for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

$$\mu'_{R' \circ S'}(x_i, y_k) = \bigvee_j ([1,1] - |\mu'_{R_i}(x_i, z_j) - \mu'_{S_i}(z_j, y_k)|)$$

Similarly, *Similarity-Min product* for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

$$\mu'_{R' \circ S'}(x_i, y_k) = \bigwedge_j ([1,1] - |\mu'_{R_i}(x_i, z_j) - \mu'_{S_i}(z_j, y_k)|)$$

Given the following notation for the resulting intervals over which the maximum or minimum is to be taken.
Then, in the context of Similarity product here, the maximum and minimum operators can be defined as follows:

\[ \gamma([I,1] - | \mu_R(x_i, z_j) - \mu_S(z_j, y_k) |) = \left[ \gamma(\mu_R^{\triangle}(z_j)), \gamma(\mu_S^{\triangle}(z_j)) \right] \]

\[ \wedge([I,1] - | \mu_R(x_i, z_j) - \mu_S(z_j, y_k) |) = \left[ \wedge(\mu_R^{\triangle}(z_j)), \wedge(\mu_S^{\triangle}(z_j)) \right] \]

The results of the application of these intervalized Similarity products to the interval membership fuzzy relations of Figs. 3 and 4 are shown respectively in Figs. 11-13.

\begin{verbatim}
Fig. 11. Resulting fuzzy relation \( R^{I_1 \odot S^{I_1}} \)
\end{verbatim}

\begin{verbatim}
\begin{array}{ccc}
 y_1 & y_2 & y_3 \\
x_1 & [0.53,0.83] & [1.00,1.00] & [0.90,0.97] \\
x_2 & [0.87,1.00] & [0.60,0.77] & [0.60,0.77] \\
\end{array}
\end{verbatim}

\begin{verbatim}
Fig. 12. Resulting fuzzy relation \( R^{I_0 \odot S^{I_1}} \)
\end{verbatim}

\begin{verbatim}
\begin{array}{ccc}
 y_1 & y_2 & y_3 \\
x_1 & [0.70,1.00] & [1.00,1.00] & [0.90,1.00] \\
x_2 & [0.90,1.00] & [0.80,0.90] & [0.80,1.00] \\
\end{array}
\end{verbatim}

\begin{verbatim}
Fig. 13. Resulting fuzzy relation \( R^{I_0 \odot S^{I_1}} \)
\end{verbatim}

\begin{verbatim}
\begin{array}{ccc}
 y_1 & y_2 & y_3 \\
x_1 & [0.30,0.60] & [1.00,1.00] & [0.90,0.90] \\
x_2 & [0.80,1.00] & [0.40,0.60] & [0.30,0.60] \\
\end{array}
\end{verbatim}

\section*{Intervalized Star Products}

Drawing upon the similar argument for the rationale behind the definitions of Similarity product and its variations for fuzzy relations with interval membership degrees, the Star product can be defined and its variations for interval membership fuzzy relations as follows.

\begin{verbatim}
Star product for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

\[ \mu_{R^{I_1 \odot S^{I_1}}}(x_i, y_k) = \frac{\sum_j (\mu_R(x_i, z_j) \mu_S(z_j, y_k))}{\sum_j} \]

Again, the trick is to define an appropriate interpretation of the so-called combined strength for interval membership degrees, which the Star product tries to determine.

Based on the definitions given for the two intervals in the previous Section, the normal interval multiplication can be defined as follows:

\[ \mu_R^{\triangle}(x_i, z_j) \mu_S^{\triangle}(z_j, y_k) = \left[ \min(\mu_R^{\triangle}(x_i, z_j)), \min(\mu_S^{\triangle}(z_j, y_k)) \right] \]

\[ \max(\mu_R^{\triangle}(x_i, z_j)), \max(\mu_S^{\triangle}(z_j, y_k)) \]

\begin{verbatim}
\begin{array}{ccc}
 y_1 & y_2 & y_3 \\
x_1 & [0.30,0.60] & [1.00,1.00] & [0.90,0.90] \\
x_2 & [0.80,1.00] & [0.40,0.60] & [0.30,0.60] \\
\end{array}
\end{verbatim}

At this point, the average can then be taken as in the case of the Similarity product.

Similarly, the Max and Min variations of the Star product can be defined below.

\begin{verbatim}
Star-Max product for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

\[ \mu_{R^{I_1 \odot S^{I_1}}}(x_i, y_k) = \left[ \mu_R^{\triangle}(x_i, z_j) \right] \mu_S^{\triangle}(z_j, y_k) \]

\[ \left[ \mu_R^{\triangle}(x_i, z_j) \right] \mu_S^{\triangle}(z_j, y_k) \]

\begin{verbatim}
\begin{array}{ccc}
 y_1 & y_2 & y_3 \\
x_1 & [0.30,0.60] & [1.00,1.00] & [0.90,0.90] \\
x_2 & [0.80,1.00] & [0.40,0.60] & [0.30,0.60] \\
\end{array}
\end{verbatim}

In the same fashion, Star-Min product for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

\[ \mu_{R^{I_1 \odot S^{I_1}}}(x_i, y_k) = \left[ \mu_R^{\triangle}(x_i, z_j) \right] \mu_S^{\triangle}(z_j, y_k) \]

In the same fashion, Star-Min product for interval membership fuzzy relations can be defined in terms of membership degree calculation as:

\[ \mu_{R^{I_1 \odot S^{I_1}}}(x_i, y_k) = \left[ \mu_R^{\triangle}(x_i, z_j) \right] \mu_S^{\triangle}(z_j, y_k) \]
Once again, the very same problem of how to interpret the maximum and minimum operators in the context of combined strength as defined by the Star product emerges. First, the resulting intervals are defined once again over which the maximum and minimum are to be taken.

\[
(\mu_{R^i}(x_j, z_j) \mu_{S^i}(z_j, y_j)) = [I_{R^i \times S^i}(z_j), h_{R^i \times S^i}(z_j)]
\]

Now, similarly defining the maximum and minimum operators for the Star product as shown below:

\[
\vee_j(\mu_{R^i}(x_j, z_j) \mu_{S^i}(z_j, y_j)) = [\vee_j(I_{R^i \times S^i}(z_j)), \vee_j(h_{R^i \times S^i}(z_j))]
\]

\[
\wedge_j(\mu_{R^i}(x_j, z_j) \mu_{S^i}(z_j, y_j)) = [\wedge_j(I_{R^i \times S^i}(z_j)), \wedge_j(h_{R^i \times S^i}(z_j))]
\]

The results from the applications of these intervalized Star products to the same interval membership fuzzy relations are shown in Figs. 14-16, respectively.

**Usage Example**

The intervalized versions of the products allows the information to be better captured in terms of interval membership degrees, than in terms of single point/value membership degrees. An example, a pair of students from two different schools (from sets \(X\) and \(Y\), respectively) based on their proficiency levels on three subjects in a set of subjects \(Z\). The proficiency level of each subject is measured by a few tests administered to each individual student. This results in a confidence interval that can be interpreted as an interval membership degree. The objective is to pair up in the best way possible, students that have similar levels of proficiency across the three subjects, so that special tutorials can be conducted for the pair, so they may progress at the same pace.

For the sake of illustration, using the data shown in Figures 3 and 4 to represent the confidence intervals. Then, looking at the results of the intervalized Similarity product as shown in Figure 11, it is obvious that what is needed for the pair of students’ \(x_1\) with \(y_2\) and \(x_2\) with \(y_1\). Assuming further, that for an unforeseen reason, \(x_1\) with \(y_1\) are not available. This then leaves pair \(x_2\) with either \(y_2\) or \(y_3\). The choice is not so obvious now since both pairs have the same resulting interval membership degree representing the similarity. Needing to look further to the results of the intervalized Similarity-Max and Similarity-Min products. It can be observed that the resulting intervals of pair \((x_2, y_2)\) appears to be more coherent than that of pair \((x_2, y_3)\), i.e. Both max and min intervals are closer to the average interval. With this additional information, the decision can be intelligently made in favor of the pair \((x_2, y_2)\). If then, rules similar to those suggested in [14] can be used to represent the kind of (meta) knowledge outlined above. A rule-based system can be constructed to perform a post-processing of the results of the products. An analysis in a similar fashion can also be conducted on the results of the intervalized Star products. Furthermore, these two set of products may be applied creatively.
in any combination to derive a desired analysis in a given application domain.

**Conclusion**

The proposed extension of the *Similarity* and *Star* products together with that of their variations to accommodate interval membership fuzzy relations has open up new horizons in information processing/analysis by means of fuzzy relations. The products can be used individually, or in combination depending on the desired semantics as dictated by the objective(s) of the processing/analysis being performed. Applications of these products to different application domains are yet to be explored and definitely worth further investigation.

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