BER Analysis for Synchronous All-Optical CDMA LANs with Modified Prime Codes

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Abstract

The analysis of the BER performance for synchronous all-optical code-division multiple-access (CDMA) local area networks (LANs) with modified prime codes is presented. The mean and variance of periodic cross-correlation values of modified prime codes are derived analytically. The effects of shot noise, thermal noise, interference and the architecture of optical receivers are included in the evaluation of the BER. It is shown that an equation of BER for ideal synchronous optical CDMA LANs previously published can be derived from our equation.

Keywords: Optical communications, code-division multiple-access, synchronous optical CDMA LANs, BER analysis, modified prime codes.

Introduction

Optical code-division multiple-access (CDMA) is one of the multiple access techniques that allows many users to share a single transmission channel through the assignment of unique signature sequences. Depending on the requirement of time synchronization, there are asynchronous or synchronous systems (Kwong et al. 1991). Comparing with asynchronous CDMA network, synchronous CDMA that requires network access among all users be synchronized can provide higher throughput (i.e. more successful transmission) and accommodate more subscribers. Recently, a synchronous all-optical CDMA LANs using modified prime sequences, which are obtained from prime sequences (Sharr and Davis 1983) of length \( N = p^2 \) and weight \( W = p \) (\( p \) is a prime number) has been proposed (Kwong et al. 1991).

The BER performance of modified prime codes in all-optical CDMA systems has been investigated by Kwong et al.(1991) and Walle and Killat (1995). However, in those works only the interference effect on the desired signal was investigated and the power loss due to the parallel architecture of the correlator receiver was not considered. In this paper we present the analysis of the BER performance for all-optical CDMA LANs with modified prime codes. In those systems the correlator receiver is of parallel optical delay line architecture (Kwong et al.1991) and the photo detector is a PIN. We investigate the effects of shot noise, thermal noise, interference and the optical loss due to the architecture of the correlator receiver on the BER performance. For evaluating the interference, the probability distribution function of periodic cross-correlation at zero time shift of modified prime codes is determined. We use the Gaussian approximation method for establishing the equation of BER performance, which is a function of the received chip optical power and the number of simultaneous users. It is shown that the equation of BER for ideal synchronous optical CDMA LANs presented in (Kwong et al.1991) can be derived from our equation. We also calculate the BER performance for some sets of modified prime sequences and show that for a given number of simultaneous users we can determine the minimum required received chip optical power, the minimum prime number \( p \), hence, the minimum sequence length in order to achieve BER=10^{-9}. 
Modified Prime Codes

A prime sequence (Sharr and Davis 1983) \( S_i = \{s_{i,0}, s_{i,1}, \ldots, s_{i,p-1}\} \) is constructed by \( s_{i,j} = i \cdot j \mod p \) where \( i,j \in GF(p) \), the Galois field and \( p \) is a prime number. The binary prime sequences are generated by mapping each prime sequence \( S_i \) to a binary code sequence \( C_i = \{c_{i,0}, c_{i,1}, \ldots, c_{i,p-1}\} \) of length \( N=p^2 \) according to

\[
C_i = \begin{cases} 
1 & \text{if } s_{i,j} \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

for \( j \neq i \).

Table 1 shows the prime sequences \( S_i \) and binary prime sequences \( C_i \) in GF(5). Note that a binary prime sequence \( C_i \) is made up of \( p \) subsequences of length \( p \) that consists of only one “1” chip and the value of each \( s_{i,j} \) in the prime sequence \( S_i \) represents the position of the “1” chip in the \( j \)th subsequence. A binary prime code sequence is approximately orthogonal (i.e., has low cross-correlation) with the code sequences of other users, but the number of code sequences is limited to the prime number \( p \), and therefore so is the number of total subscribers. A scheme was proposed which can accommodate a greater number of subscribers for the same bandwidth-expansion, at the expense of requiring synchronization. This scheme uses a set of code sequences generated from time-shifted version of binary prime code sequences. The new code is called modified prime code (Kwong et al., 1991).

<table>
<thead>
<tr>
<th>( i )</th>
<th>Prime Sequence ( S_i )</th>
<th>Binary prime code sequences ( C_i )</th>
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<tr>
<td>0</td>
<td>00000</td>
<td>( C_0 = 0000,0000,0000,0000,10000,10000,10000,10000 )</td>
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<tr>
<td>1</td>
<td>01234</td>
<td>( C_1 = 0000,01000,00010,00001,00001 )</td>
</tr>
<tr>
<td>2</td>
<td>02413</td>
<td>( C_2 = 0000,00100,00001,01000,00010 )</td>
</tr>
<tr>
<td>3</td>
<td>03142</td>
<td>( C_3 = 0000,00010,01000,00001,00100 )</td>
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<tr>
<td>4</td>
<td>04321</td>
<td>( C_4 = 0000,00001,00100,00010,01000 )</td>
</tr>
</tbody>
</table>

Table 1. Binary prime sequences in GF(5)

In order to generate modified prime code sequences, each of the original \( p \) prime sequence \( S_i \) is taken as a seed. The code sequences of the first group (i.e., \( i = 0 \)) are obtained by left-rotating the binary prime code sequence \( C_0 \). \( C_0 \) can be left-rotated \( p-1 \) times before being recovered, so that \( p-1 \) new code sequences can be generated from \( C_0 \). For other \( p-1 \) groups (i.e., \( i = 1, \ldots, p-1 \)), the elements of the corresponding prime sequence \( S_i \) can be left-rotated \( p-1 \) times to create new prime sequences \( S_{it} = (S_{it0}, S_{it1}, \ldots, S_{itp-1}) \), where \( t \) \((0 < t \leq p-1) \) represents the number of times \( S_i \) has been left-rotated. Each prime sequence \( S_{it} \) is then mapped into a modified prime code sequence \( C_{it} \). Therefore, a total of \( p \) modified prime sequences per group are obtained. In Table 3 each modified prime sequence \( C_{it0} \) is the same as the original binary prime code sequence \( C_i \), whereas the other modified prime sequences \( C_{it} \) are time-shifted version of \( C_i \). Since the number of possible subscribers is determined by the number of code sequences, in synchronous CDMA system, \( p^2 \) subscribers can be supported, a factor of \( p \) larger than for asynchronous CDMA. These \( p^2 \) sequences are still pseudo-orthogonal to each other. The autocorrelation peak for all new code sequences at each decoder output has an amplitude equal to \( p \) and is made to occur in the last chip position to which all receivers are synchronized. For a synchronous system, the amplitude of the periodic cross-correlation at zero-time shift is 0 if the two modified prime sequences originate from the same group and is 1 if they originate from different groups (Kwong et al, 1991). If \( C_{it} \) and \( C_{jk} \) are modified prime sequences the periodic cross-correlation at zero-time shift can be written as

Transmitter and Receiver

Figs. 1 and 2 show the block diagrams of the transmitter and receiver for synchronous optical CDMA networks using modified prime sequences, where parallel architecture is used for sequence generation, selection and correlation (Kwong et al. 1991). The transmitter for generating sequences of length \( N = p^2 \) consists of a \( 1 \times N \) optical power splitter, a set of \( N \) parallel optical fiber delay lines, and a \( N \times l \) power
A laser produces high intensity pulse streams at the data rate. The duration of each pulse is less than or equal the chip time $T_c = T/N$, where $T$ is the bit time. The time of emitting the light pulses is controlled by the received clock, so that a pulse is emitted only at the beginning of each bit time. These pulses are modulated by an optical gate, such as a directional coupler switch, which is driven by the data waveform to generate the optical data signal. At the $1xN$ optical power splitter, the optical pulse is splitted into $N$ pulses which are then delayed in the set of parallel optical fiber delay lines. The sequence selection can be realised by controlling $N$ opto-electrical switches each of which is located in one branch of the transmitter. The switch in a given branch is closed if this branch corresponds to the position of a "1" chip in the selected sequence. This switch is opened if the branch corresponds to the position of a "0" chip. By controlling these switches, only $p$ (the number of “1” chips in a sequence) properly delayed pulses according to the position of "1’s" in the intended receiver address sequence are selected and recombined so that the resulting optical sequence is obtained at the output of the optical combiner and is transmitted to the network. Clearly, this transmitter can be programmed to optically generate any of the $p^2$ modified prime sequences of length $N$. However, in order to retain addressing programmability the number of delay lines, coupler branches, and opto-electrical switches must equal $N$, thereby, placing serious limits on the sequence length $N$.

Fig. 1. Optical transmitter

In the receiver, the decoder receives optical sequences from the network. The decoder consists of $N$ optical fiber delay lines connected in parallel using an $IxN$ optical power splitter and a $NxI$ optical power combiner. In this receiver, the selection of delay lines is such that the delays correspond to the position of "1's" in the time reversed version of the receiver reference sequence. At the input of the correlator receiver the received optical signal is split into $N$ different paths. However, only $p$ properly delayed signals are recombined at the optical combiner. At each decoder output, the auto-correlation peak has an amplitude equal to $p$ and is made to occur in the last chip position of data bit to which all receivers are synchronized. The synchronized signals are then photo-detected by a photo-detector (PIN). The data are received by threshold-detecting the auto-correlation peaks. Note that the described receiver can be programmed to correlate the received signal to any of the $p^2$ modified prime sequences of length $N$. However, only $p$ optical fiber delay lines are needed if the reference sequence of the receiver
is fixed to a given modified prime sequence. In this case the size of the optical power splitter and combiner is reduced to $1 \times p$ and $p \times 1$, respectively and $N$ opto-electrical switches are also eliminated. We can also assumed that the optical loss factor for receivers of fixed reference sequence is $S = p^2$. For fully programmable receivers, this factor would be $S = N^2 = p^2$, the same as that for the fully programmable transmitter.

![Optical receiver](image.png)

**BER Performance Analysis**

We investigate a synchronous all-optical CDMA network using modified prime codes with $K$ simultaneous users. Assume that every user transmits the same chip optical power and the received chip optical power is $P$ for all $k$ ($1 \leq k \leq K$) and the $K$ simultaneous transmitters are synchronized to each other. The total received optical signal $R(t)$ at the input of the $i$th receiver is the incoherent sum

$$ R_i(t) = \sum_{k=1}^{K} A'_k(t) B'_k(t) $$

where $A'_k(t)$ is the code waveform of the $k$th user, and $B'_k(t)$ is the transmitted binary signal. The term $B'_k(t)$ is given by

$$ B'_k(t) = B_k(n) \Pi_{T}(t) $$

where $B_k(n) \in \{0,1\}$ is the data bit and $\Pi_T(t)$ is the unit amplitude unipolar rectangular pulse of one bit time duration $T$. The waveforms $A'_k(t)$ can be written as

$$ A'_k(t) = \{A_k(n)\} $$

where $\{A_k(n)\}$ is the address sequence (or reference sequence) of the intended receiver, which is a modified prime code sequence of length $N$. $\Pi_{T}(t)$ is the unit amplitude unipolar rectangular pulse of duration $T_c = T/N$. At the $i$th station, the received optical signal is correlated to the address waveform $A'_i(t)$ and the correlator output optical signal is converted into electrical current in a PIN photodiodes of responsivity $R$ (A/W). The electrical signal at the output of the photodiode at time $t = T$ is

$$ I_{ST}(t) = \int_{0}^{T} R(t) A(t) dt $$

where $S$ is the optical loss factor due to the correlator receiver architecture, $n(t)$ is the composite noise current composed of shot noise, dark current noise and thermal noise. The current at the output of the photodiode can be also written as

$$ I_{T}(t) = S P B(t) I_{n} $$

where $B(t)$ is the data bit sending to the $i$th receiver and $B(t)$ can takes one of the two values $\{0,1\}$ with equal probability. In equation (1) the first term represents the desired signal, the second term is the total multiple-access interference (MAI) with $(RP/S)I_{k,i}^2$ representing the interference produced by the $k$th user at the $i$th receiver and the last term is the composite noise.
Interference

The random variable $I_{k,i}$ can be expressed as

$$I_{k,i} = B_k(0) A_i(n) A_k(n)$$  \hspace{1cm} (2)

where $B_k(0)$ is the interference bit emitted by the $k$-th transmitter which can take one of the two values $\{0,1\}$ with equal probability. If the interference bit is “1” Equation (2) gives the periodic cross-correlation of sequences $\{A_i(n)\}$ and $\{A_k(n)\}$ at zero time-shift. The total multiple-access interference in Equation (1) can be modeled as a gaussian process with the following mean value and variance

$$E[I_{k,i}] = \frac{1}{N} \sum_{n=0}^{N-1} A_i(n) A_k(n)$$ \hspace{1cm} (3)

$$Var[I_{k,i}] = \frac{1}{N} \sum_{n=0}^{N-1} (A_i(n) A_k(n))^2$$ \hspace{1cm} (4)

where $E[I_{k,i}]$ and $Var[I_{k,i}]$ are the mean and variance of the random variable $I_{k,i}$. For calculating those parameters, consider a receiver using the address sequence $\{A_i(n)\}$ from a set of $p^2$ modified prime sequences of length $N = p^2$. The interference bits emitted by a transmitter can take one of the two values $\{0,1\}$ with equal probability. If the interference bit is “1” and its destination is a user using the sequence $\{A_k(n)\}$ in the same group with $\{A_i(n)\}$, the periodic cross-correlation at zero-time shift of $\{A_i(n)\}$ and $\{A_k(n)\}$ is 0. Therefore, if all pairs of sequences are counted the number of all possible cross-correlations equal to 0 is

$$\text{Total number of cross-correlations equal to 0}$$

If the interference bit is “1” and its destination is a user using the sequence $\{A_k(n)\}$ in one of the other $(p-1)$ groups, the periodic cross-correlation at zero-time shift of $\{A_i(n)\}$ and $\{A_k(n)\}$ is 1. Hence, the number of cross-correlations equal to 1 is

$$\text{Total number of cross-correlations equal to 1}$$

If the interference bits is “0” the periodic cross-correlation at zero-time shift of $\{A_i(n)\}$ and the interference sequence is always 0. The number of all possible cross-correlations equal to 0 due to “0” interference bits is

$$\text{The total number of cross-correlations equal to 0 and 1 due to both “0” and “1” interference bits is}$$

$$p^2(p^2-1)$$

Hence, the probabilities $p(0)$ and $p(1)$ of having periodic cross-correlation equal to 0 and 1 are

$$p(0) = \frac{1}{2}$$

$$p(1) = \frac{1}{2}$$

The mean value and the variance of the random variable $I_{k,i}$ can be calculated from the above probabilities by using definitions of mean value and variance. The mean value is

$$\text{Mean value}$$

and the variance can be calculated by

$$\text{Variance}$$

Noise

The shot noise in the photodiode results from the randomness associated with the rate of the arrival of the photons at the photodiode and it can be approximated by gaussian statistics. The variance of the shot noise generated by a PIN can be calculated by (Agrawal 1997)

$$\text{Noise}$$

where $q = 1.602 \times 10^{-19}$ C is the electric charge, $i_m$ is the mean value of the photo-current, $i_d$ is the dark current, $B$ is the noise-equivalent receiver bandwidth. In a non-coherent optical fiber CDMA system when a "0" is transmitted to the $i$th user, at the $i$th receiver the PIN receives only the MAI signal. Therefore, the photo current consists of only the MAI current and the variance of the shot noise may be deduced from Equations (3), (5), (7) and calculated by

$$\text{Current}$$

When a "1" is transmitted to the $i$th user, at the $i$th receiver the PIN receives the desired signal and the MAI signal. Therefore, the photo current consists of the desired signal current as
well as the MAI current and the variance of the shot noise is

\[ s_i = \sum_{k=1}^{p} I_k^p \] 

(9)

The thermal noise can be also modeled as a gaussian random process and the variance of the thermal noise is

\[ s_T^2 = k_B T R_L \] 

(10)

where \( k_B = 1.38 \times 10^{-23} \) J/K is the Boltzmann’s constant, \( T \) is the receiver noise temperature and \( R_L \) is the receiver load resistor.

**BER Calculation**

The BER can be calculated by (Agrawal 1997)

\[ \text{BER} = \Phi - \frac{1}{2} \left( \frac{1}{2} \right) \] 

(11)

where \( i_0 \) and \( i_1 \) denote the average currents at the input of the detector for bits “0” and “1” respectively and \( \sigma_0 \) and \( \sigma_1 \) are the corresponding standard deviations of their gaussian distributions, \( I_{th} \) is the optimum detection threshold given by

\[ I_{th} = \frac{i_0 - i_1}{\sigma_0/\sigma_1} \] 

(12)

From the above analysis, we have

\[ i_k = \frac{1}{2} (i_0 + i_1) \] 

(13)

\[ \sigma_k = \frac{1}{2} (\sigma_0 + \sigma_1) \] 

(14)

\[ q_k = \frac{1}{2} (q_0 - q_1) \] 

(15)

\[ q_k^p = \frac{1}{2} (q_0^p + q_1^p) \] 

(16)

In the ideal synchronous optical CDMA system investigated by Kwong et al., (1991) only the interference effect on the desired signal was considered. In such a case \( RP/S = I \) and the shot noise, thermal noise are considered to be null. Under those conditions Equation (11) becomes

\[ \text{BER} = \Phi \] 

(17)

For relatively large \( p \), \( (p^2 + 2p)/(p^2 + 2p + 1) \approx 1 \), hence, Equation (17) can be written as

\[ \text{BER} = \Phi \] 

(18)

It can be seen that Equation (18) is the same as Equation (10) derived by Kwong et al., (1991).

**Numerical Results**

The BER performance for ideal synchronous optical fiber CDMA systems with modified prime codes generated by \( p=5 \) is calculated by two different methods: the method presented in this paper (Equations (17)) and that of (Kwong et al. 1991). The calculated BERs as functions of the number of simultaneous users \( K \) are showed in Figure 3. It’s can be seen that it is difficult to differentiate the two curves.

Fig.3 BER for ideal systems with MPR of p=5

Next we calculate the BER performance for systems using parallel delay-line correlator receivers of fixed address sequences (Kwong et al. 1991). The effects of all shorts of noises and interference are taken into account. Substituting the optical loss factor \( S = p^2 \) into Equations (13)-(16) we have
It is seen from Equation (12) that the optimum threshold \( I_{th} \) is not a fixed value but it depends on the number of simultaneous users \( K \). Therefore, in order to achieve the optimum BER it is required that each receiver of the network is capable of determining the optimum detection threshold. If the threshold \( I_{th} \) is set as a fixed value the BER is degraded. In our calculation we assume that every user can determine the optimum threshold \( I_{th} \), hence, the obtained results are optimum.

\[
I_{th} = -\left(1 + \left(\frac{q_B}{K\sqrt{p}}\right)\right)
\]

(19)

\[
I_{th} = -\left(1 + \left(\frac{q_B}{K\sqrt{p}}\right)\right)
\]

(20)

\[
21 + 12 + \left(\left(\frac{q_B}{K\sqrt{p}}\right)\right)
\]

(21)

\[
21 + 12 + \left(\left(\frac{q_B}{K\sqrt{p}}\right)\right)
\]

(22)

**Table 2. Typical link parameter**

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIN responsivity</td>
<td>( R )</td>
<td>0.78 A/W</td>
</tr>
<tr>
<td>PIN dark current</td>
<td>( i_D )</td>
<td>1 nA</td>
</tr>
<tr>
<td>Receiver load resistor</td>
<td>( R_L )</td>
<td>50Ω</td>
</tr>
<tr>
<td>Receiver noise temperature</td>
<td>( T_T )</td>
<td>300°C</td>
</tr>
</tbody>
</table>

The BER is calculated based on Equations (11), (19)-(22). The data bit rate is 10 Mb/s, the number of simultaneous users is \( K=10 \), the received chip optical power is in the range \( P = \{-20 \text{ dBm}, 10 \text{ dBm}\} \) and other parameters are shown in Table 2 (Agrawal 1997). The obtained results are presented in Figure 4 for \( p=13, 17, 19 \) and 37. It is seen that for low received chip optical power \( P \), systems with smaller \( p \) cause lower BER. This is because in those systems the optical loss due to the parallel delay line correlator receiver is smaller. However, for a number of simultaneous users \( K =10 \), BER=10\(^{-9}\) can only be achieved in systems with \( p \geq 19 \). For systems with \( p < 19 \), even for an arbitrary high received chip optical power \( P_s \) we never can have BER=10\(^{-9}\). We can also determine the minimum required received chip optical power \( P \) and the minimum value of \( p \), hence, the minimum sequence length \( N \) (because \( N=p^2 \)) in order to achieve BER=10\(^{-9}\). For example, from Figure 4, if we want to have BER=10\(^{-9}\) for \( K=10 \) the minimum \( P \) is about –2 dBm and the minimum sequence length \( N \) (for \( p = 19 \)).

**Fig. 4. BER for systems with \( K=10 \), \( p=13, 17, 19 \) and 37**

**Conclusion**

In this paper the analysis of the BER performance for non-coherent synchronous all-optical CDMA systems with modified prime codes is presented. Using the gaussian approximation method we establish the general equation of the BER as a function of the received chip optical power and the number of simultaneous users. The effects of all shorts of noise and interference are taken into account. We show that the equation of BER for ideal synchronous CDMA systems with modified prime codes (Kwong et al.1991) is a special case of our equation. Finally, we calculate the BER performance for real systems taken into account the effect of shot noise, thermal noise, interference and the architecture of the correlator receiver. We show that for a given number of simultaneous users \( K \) we can determine the minimum required received chip optical power \( P \) and the minimum value of \( p \), hence, the minimum sequence length \( N \) in order to achieve BER=10\(^{-9}\).
References


Table 3. Modified prime sequences $C_{i,t}$ in GF(5)

<table>
<thead>
<tr>
<th>Group i</th>
<th>New prime sequences $S_{i,t}$</th>
<th>Modified prime sequence $C_{i,t}$</th>
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<td>0</td>
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<td>$C_{00} = 10000,10000,10000,10000,10000$</td>
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<td>22222</td>
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